After the proof of the Bloch-Kato conjecture, we know that the $\mathbb{F}_p$-cohomology ring $H^\bullet(G, \mathbb{F}_p)$ of a maximal pro-$p$ Galois group $G$ is a quadratic algebra. Recently L. Positselsky conjectured that such ring is a quadratic Koszul algebra – and he proved it is for local and global fields. We prove this conjecture for the class of pro-$p$ groups of elementary type, and for such pro-$p$ groups the quadratic (or Koszul) dual of $H^\bullet(G, \mathbb{F}_p)$ is a canonical graded algebra induced by the complete group algebra $\mathbb{F}_p[G]$. Moreover, we prove that for any maximal pro-$p$ Galois group $G$ the quadratic dual of $H^\bullet(G, \mathbb{F}_p)$ is the “quadratic cover” of such graded algebra, which carries also some arithmetic information. This is a joint work with J. Mináč and N.D. Tân.