THE (INTEGRAL) ISOMORPHISM PROBLEM, A (GEOMETRIC)
GROUP THEORY POINT OF VIEW

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Abstract. Given a finite group $G$ and ring $R$ it is natural to ask which
properties of $G$ are determined by its representations over $R$. The integral
isomorphism problem asserts that $G$ is fully determined by its integral group
ring $\mathbb{Z}G$. False in general, it holds however for 'most' solvable and simple
groups. In this talk, instead of the typical ring or module theoretical approach,
we will look at above problem from a geometric group theory point of view.
The question can namely be viewed as "how rigid is $G$ inside $\mathcal{U}(\mathbb{Z}G)$?", where
the latter is a finitely presented (arithmetic) group.

More precisely, after an introductory part, we will discuss the question
when $\mathcal{U}(\mathbb{Z}G)$ and $\text{GL}_n(\mathcal{O})$ for $\mathcal{O}$ an order in a division algebra have property
($\mathcal{F}A$). This property holds if for any action on an arbitrary (simplicial) tree
there is a vertex fixed by all elements of the acting group. This is equivalent
to having finite abelianisation and admitting no non-trivial decomposition as
amalgamated product. In the last part of the talk, if time allows, we mention
some questions and results concerning concrete constructions of amalgamated
products inside $\mathcal{U}(\mathbb{Z}G)$. All results presented are joint works with Andreas
Bächle, Eric Jespers, Ann Kiefer and Doryan Temmerman.