Let's consider a triangulation of the triangle by $V-3$ interior vertices, with a total $V$ vertices, $E$ edges and $F$ small triangles. Euler's formula says $V - E + F = 1$, from which the inequality $E \leq 3V - 6$ can be concluded.

What are the analogous equalities and inequalities for triangulations of the tetrahedron (= a triangular pyramid)? And for higher dimensions?

Discussion of these questions will lead us to polyhedra, linear algebra, rings, topology, and more.