On the horseshoe conjecture for maximal distance minimizers

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We study the properties of sets Σ having the minimal length over the class of closed connected sets Σ ⊂ \mathbb{R}^2 satisfying the inequality on energy

\[ F_M(Σ) := \max_{y \in M} \text{dist} \ (y, Σ) \leq r \]

for a given compact set \( M \subset \mathbb{R}^2 \) and some given \( r > 0 \).

Let \( M \) be a closed convex curve with the minimal radius of curvature \( R > r \). Then the connected curve Σ is called a horseshoe, if \( F_M(Σ) = r \) and Σ is a union of an arc \( q \) of \( M_r \) (the shift of \( M \) on \( r \) toward the inner normal direction) with two tangent segments to \( M_r \) in the different ends of \( q \) (see Figure 1).

We prove that for every closed convex curve \( M \) with the minimal radius of curvature \( R \) and for every \( r < R/5 \) the set of minimizers contains only horseshoes.

Hereby we prove the conjecture of Miranda, Paolini and Stepanov describing the set of minimizers for \( M \) a circumference of radius \( R > 0 \) for the big enough ratio \( R/r \).

![Figure 1: A horseshoe.](image-url)