Regularity of solutions of Hamilton-Jacobi equation on a domain

by

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In this lecture, we will explain a new method to show that regularity on the boundary of a domain implies regularity in the inside for PDE’s of the Hamilton-Jacobi type.

The method can be applied in different settings. One of these settings concerns continuous viscosity solutions $U : \mathbb{T}^N \times [0, +\infty[ \to \mathbb{R}$ of the evolutionary equation

$$\partial_t U(x, t) + H(x, \partial_x U(x, t)) = 0,$$

where $\mathbb{T}^N = \mathbb{R}^N / \mathbb{Z}^N$, and $H : \mathbb{T}^N \times \mathbb{R}^N$ is a Tonelli Hamiltonian, i.e. $H(x, p)$ is $C^2$, strictly convex superlinear in $p$.

Let $D$ be a compact smooth domain with boundary $\partial D$ contained in $\mathbb{T}^N \times [0, +\infty[$. We show that if $U$ is differentiable at each point of $\partial D$, then this is also the case on the interior of $D$.

There are several variants of this result in different settings.

To make the result accessible to the layman, we will explain the method on the function distance to a closed subset of an Euclidean space. This example contains all the ideas of the general case.