On Semi-linear elliptic systems exhibiting critical behavior

Lorena Aguirre Salazar

Research Master Thesis under the direction of Professor Itai Shafrir

Abstract

For $N \geq 3$ and $m$ a positive integer, let $A = [a_{ij}]$ be an $m \times m$ matrix with nonnegative entries and define $p^* := \frac{N}{N-2}$. We are interested in studying the following semilinear elliptic system

$$
\begin{cases}
-\Delta u_i = \left(\sum_{j=1}^{m} a_{ij}u_j + c_i\right)^{p^*}_+ & \text{in } \Omega, \\
\int_{\Omega} \left(\sum_{j=1}^{m} a_{ij}u_j + c_i\right)^{p^*}_+ = M_i,
\end{cases}
$$

(S)

with boundary conditions

$$u_i = 0 \quad \text{on } \partial\Omega, \quad (\text{if } \Omega \text{ is bounded}) \quad (\text{BC})$$

for $i = 1, \cdots, m$, where the vector of positive masses $\bar{M} = (M_i) \in (\mathbb{R}_+)^m$ is given, $\Omega \subseteq \mathbb{R}^N$ is either a bounded smooth domain or the whole space $\mathbb{R}^N$, and $\bar{c} = (c_i) \in \mathbb{R}^m$ is an unprescribed vector of constants.

Our work is composed of four sections consisting of

- proving some general results about solutions to our problem,
- finding conditions under which each component of a solution to (S) in $\mathbb{R}^N$ is radially symmetric and decreasing about some point,
- introducing a dual formulation of the problem in bounded smooth domains that enables us to prove existence of solutions by minimization, and,
- detailing the relation between existence of solutions to the problem in a ball and existence of solutions in $\mathbb{R}^N$. 