Mathematical models of hysteresis phenomena

A. Visintin - Trento

Hysteresis may be defined as a rate-independent memory effect, and occurs in several phenomena in physics and other disciplines. It was represented via continuous hysteresis operators in the 1970s by Krasnosel’skiĭ and co-workers [KrPo]. Some models of hysteresis in elasto-plasticity may be represented via variational inequalities, and may thus be dealt within the framework of convex analysis. Others require a different approach.

Partial differential equations with hysteresis have been considered since the early 1980s. They include quasilinear parabolic and hyperbolic equations of the form

$$\frac{\partial}{\partial t} [u + F(u)] + Au = f, \tag{1}$$

$$\frac{\partial^2}{\partial t^2} [u + F(u)] + Au = f. \tag{2}$$

Here $F$ denotes a continuous hysteresis operator, and $A$ is a linear elliptic operator. A suitable monotonicity-type condition is assumed for $F$. The Cauchy problem for (1) has one and only one solution [Vi1,Vi3].

A different approach is in order in case of discontinuous hysteresis, e.g. for the simple (delayed) relay operator, which is characterized by a rectangular hysteresis loop. Although this is single relation, it can be represented as a system of two variational inequalities. Existence and uniqueness of a weak solution for the problem (1) was proved also in this case.

The Cauchy problem for the hyperbolic equation (2) also has a weak solution [Vi2, Vi3]. This rests upon the regularization effect of hysteresis.

Details may be found in the monograph [Vi1] and the survey [Vi3]. A different approach is addressed in [MiRo].

References


