A SIMPLE CLOSED FORM FOR TRIANGULAR MATRIX POWERS

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Abstract. Given a $k \times k$ triangular matrix $M = [m_{i,j}]$ with unique diagonal elements, a simple recursive formula is used to define a set of $(k+2)/2$ power factors, $p_{i,j,s}$, which are independent of the power to which the matrix is raised. Then for any power of $M$, negative, zero or positive (positive only, if the matrix is singular), the $(i,j)$-th element of $M^n$ is given by a linear combination of power factors and powers of diagonal elements, namely $m_{i,j} = \sum_{s=1}^{(k+2)/2} p_{i,j,s} m_{s,s}^{n-1}$.

Key words. Matrix, Triangular, Powers, Closed form.

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