FAST LOW RANK APPROXIMATIONS OF MATRICES AND TENSORS

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Abstract. In many applications such as data compression, imaging or genomic data analysis, it is important to approximate a given $m \times n$ matrix $A$ by a matrix $B$ of rank at most $k$ which is much smaller than $m$ and $n$. The best rank $k$ approximation can be determined via the singular value decomposition which, however, has prohibitively high computational complexity and storage requirements for very large $m$ and $n$.

We present an optimal least squares algorithm for computing a rank $k$ approximation to an $m \times n$ matrix $A$ by reading only a limited number of rows and columns of $A$. The algorithm has complexity $O(k^2 \max(m,n))$ and allows to iteratively improve given rank $k$ approximations by reading additional rows and columns of $A$. We also show how this approach can be extended to tensors and present numerical results.

Key words. Singular value decomposition, CUR decomposition, Rank $k$ approximation, Least squares, Tucker decomposition.

AMS subject classifications. 15A18, 15A69, 65F15, 93E24.