STRUCTURE OF NILPOTENT MATRICES OVER FIELDS

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Abstract. A zero-nonzero pattern $A$ is said to be potentially nilpotent over a field $F$ if there exists a nilpotent matrix with entries in $F$ having zero-nonzero pattern $A$. We explore the construction of potentially nilpotent patterns over a field. We present classes of patterns which are potentially nilpotent over a field $F$ if and only if the field $F$ contains certain roots of unity. We then introduce some sparse patterns of order $n \geq 4$ which are spectrally arbitrary over $\mathbb{C}$ but not over $\mathbb{R}$. We also identify all irreducible patterns of order four which are potentially nilpotent over $\mathbb{R}$ or $\mathbb{C}$.

Key words. Nonzero pattern, Spectrum, Potentially nilpotent, Spectrally arbitrary, Nilpotent-Jacobian method.

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