DECOMPOSITION OF A RING INDUCED BY MINUS PARTIAL ORDER∗

DRAGAN S. RAKIĆ†

Abstract. The minus partial order linear algebraic methods have proven to be useful in the study of complex matrices. This paper extends study of minus partial orders to general rings. It is shown that the condition \( a <^-b \), where \( <^- \) is the minus partial order, defines two triples of orthogonal idempotents, and thus, a decomposition of a ring into a direct sum of abelian groups. Hence, several well-known results concerning minus partial order on real and complex matrices are generalized. Analogous decompositions for rectangular matrices over a ring and for Banach space operators are obtained. The equivalent conditions for the invariance of \( ab^{(1)}a \) under the choices of \( b^{(1)} \) are also obtained. An original inspiration for this work came from the study of minus partial order on complex matrices and from linear algebra methods.

Key words. Minus partial order, Generalized inverse, Idempotent, Von Neumann regular ring, Operator matrices, Peirce decomposition.

AMS subject classifications. 15A09, 06A06, 16U99.

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†Faculty of Sciences and Mathematics, University of Niš, Višegradska 33, PO Box 224, 18000 Niš, Serbia (rakic.dragan@gmail.com). Supported by the Ministry of Education and Science, Government of the Republic of Serbia, grant no. 174007.