AN EXTENSION OF THE CLASS OF MATRICES ARISING IN THE NUMERICAL SOLUTION OF PDES

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Abstract. This paper studies block matrices \( A = [A_{ij}] \in \mathbb{C}^{km \times km} \), where every block \( A_{ij} \in \mathbb{C}^{k \times k} \) for \( i, j \in (m) = \{1, 2, \ldots, m\} \) and \( A_{ii} \) is non-Hermitian positive definite for all \( i \in (m) \). Such a matrix is called an extended \( H \)–matrix if its block comparison matrix is a generalized \( M \)–matrix. Matrices of this type are an extension of generalized \( M \)–matrices proposed by Elsner and Mehrmann [L. Elsner and V. Mehrmann. Convergence of block iterative methods for linear systems arising in the numerical solution of Euler equations. Numer. Math., 59:541–559, 1991.] and generalized \( H \)–matrices by Nabben [R. Nabben. On a class of matrices which arise in the numerical solution of Euler equations. Numer. Math., 63:411–431, 1992.]. This paper also discusses some properties including positive definiteness and invariance under block Gaussian elimination of a subclass of extended \( H \)–matrices, especially, convergence of some block iterative methods for linear systems with such a subclass of extended \( H \)–matrices. Furthermore, the incomplete \( LDU \)–factorization of these matrices is investigated and applied to establish some convergent results on some iterative methods. Finally, this paper generalizes theory on generalized \( H \)–matrices and answers the open problem proposed by R. Nabben.

Key words. Extended \( H \)–matrices, Generalized \( M \)–matrices, Generalized \( H \)–matrices.

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