INCIDENCE MATRIX AND COVER MATRIX OF NESTED INTERVAL ORDERS

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Abstract. For any poset $P$, its incidence matrix $\mathcal{I}$ and its cover matrix $\mathcal{C}$ are the $P \times P$ $(0, 1)$ matrices such that $\mathcal{I}(x, y) = 1$ if and only if $x$ is less than $y$ in $P$ and $\mathcal{C}(x, y) = 1$ if and only if $x$ is covered by $y$ in $P$. It is shown that $\mathcal{I}$ and $\mathcal{C}$ are conjugate to each other in the incidence algebra of $P$ over a field of characteristic 0 provided $P$ is the nested interval order. In particular, when $P$ is the Bruhat order of a dihedral group, which consists of a special family of nested intervals, $\mathcal{I}$ and $\mathcal{C}$ turn out to be conjugate in the incidence algebra over every field. Moreover, $\mathcal{I}$ and $\mathcal{C}$ are proved to be conjugate in the incidence algebra over every field when $P$ is the weak order of a dihedral group. Many relevant problems and observations are also presented in this note.

Key words. Hierarchy, Jordan canonical form, Rank, Strict incidence algebra.

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