GUT-MAJORIZATION AND ITS LINEAR PRESERVERS

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Dedicated to the memory of Batool Bagheri, the founder of mathematics house of Kerman.

Abstract. Let $M_{n,m}$ be the set of all $n \times m$ real matrices. An $n \times m$ matrix $R = [r_{ij}]$ is $g$-row stochastic if $\sum_{k=1}^{m} r_{ik}$ is equal to 1 for all $i$ $(1 \leq i \leq n)$. Let $X, Y \in M_{n,m}$. The matrix $X$ is said to be gut-majorized by $Y$ (denoted by $X \preceq_{gut} Y$), if there exists an $n \times n$ upper triangular $g$-row stochastic matrix $R$ such that $X = RY$. Recall that a linear function $T : M_{n,m} \to M_{n,m}$ preserves (or strongly preserves) a relation $\sim$, if $TX \sim TY$ whenever $X \sim Y$ (or $TX \sim TY$ if and only if $X \sim Y$). The present papers establishes some facts about $\preceq_{gut}$ on $M_{n,m}$, and characterizes the structure of all (strong) linear preservers of $\preceq_{gut}$ on $M_{n,m}$.

Key words. Linear preserver, Strong linear preserver, Gut-Majorization, $g$-Row stochastic matrix.

AMS subject classifications. 15A04, 15A21.

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