THE CHARACTERISTIC SET WITH RESPECT TO K-MAXIMAL VECTORS OF A TREE∗

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Abstract. Let T be a tree on n vertices and L(T) be its Laplacian matrix. The eigenvalues and eigenvectors of T are respectively referred to those of L(T). With respect to a given eigenvector Y of T, a vertex u of T is called a characteristic vertex if Y[u] = 0 and there is a vertex w adjacent to u with Y[w] ≠ 0; an edge e = (u, w) of T is called a characteristic edge if Y[u]Y[w] < 0. C(T, Y) denotes the characteristic set of T with respect to the vector Y, which is defined as the collection of all characteristic vertices and characteristic edges of T corresponding to Y.

Let λ₁(T) ≤ λ₂(T) ≤ · · · ≤ λₙ(T) be the eigenvalues of a tree T on n vertices. An eigenvector is called a k-vector (k ≥ 2) of T if the eigenvalue λₖ(T) associated by this eigenvector satisfies λₖ(T) > λₖ−₁(T). The k-vector Y of T is called k-maximal if C(T, Y) has maximum cardinality among all k-vectors of T. In this paper, the characteristic set with respect to any k-maximal vector of a tree is investigated by exploiting the relationship between the cardinality of the characteristic set and the structure of this tree. With respect to any k-maximal vector Y of a tree T, the structure of the trees T satisfying |C(T, Y)| = k − 1 − t for any t (0 ≤ t ≤ k − 2) are characterized.

Key words. Laplacian matrix, Characteristic set, k-Vector, k-Maximal vector.

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