SCALING PROPERTIES OF PATHS ON GRAPHS

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Abstract. Let $G$ be a directed graph on finitely many vertices and edges, and assign a positive weight to each edge on $G$. Fix vertices $u$ and $v$ and consider the set of paths that start at $u$ and end at $v$, self-intersecting in any number of places along the way. For each path, sum the weights of its edges, and then list the path weights in increasing order. The asymptotic behaviour of this sequence is described, in terms of the structure and type of strongly connected components on the graph. As a special case, for a Markov chain the asymptotic probability of paths obeys either a power law scaling or a weaker type of scaling, depending on the structure of the transition matrix. This generalizes previous work by Mandelbrot and others, who established asymptotic power law scaling for special classes of Markov chains.

Key words. Non-negative matrices, Perron-Frobenius theory, Directed graphs, Markov chains, Power law scaling.

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