RESULTS AND OPEN PROBLEMS IN MATCHINGS IN REGULAR GRAPHS

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Abstract. This survey paper deals with upper and lower bounds on the number of $k$-matchings in regular graphs on $N$ vertices. For the upper bounds we recall the upper matching conjecture which is known to hold for perfect matchings. For the lower bounds we first survey the known results for bipartite graphs, and their continuous versions as the van der Waerden and Tverberg permanent conjectures and its variants. We then discuss non-bipartite graphs. Little is known beyond the recent proof of the Lovász-Plummer conjecture on the exponential growth of perfect matchings in cubic bridgeless graphs. We discuss the problem of the minimum of hafnians on the convex set of matrices, whose extreme points are the adjacency matrices of subgraphs of the complete graph corresponding to perfect matchings. We also consider infinite regular graphs. The analog of $k$-matching is the $p$-monomer entropy, where $p \in [0,1]$ is the density of the number of matchings.

AMS subject classifications. 05C30, 05C70, 15A15.

Key words. Hafnians, permanents, doubly stochastic matrices, matching polytope of a complete graph, bipartite graphs, non-bipartite graphs.

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