ON GRADED MATRIX HOM-ALGEBRAS∗

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Abstract. Consider an (associative) matrix algebra \(M_I(R)\) graded by means of an abelian group \(G\), and a graded automorphism \(\phi\) on \(M_I(R)\). By defining a new product by \(x \star y := \phi(x)\phi(y)\) on \(M_I(R)\), \((M_I(R), \star)\) becomes a hom-associative algebra graded by a twist of \(G\). The structure of \((M_I(R), \star)\) is studied, by showing that \(M_I(R)\) is of the form

\[
M_I(R) = U + \sum_j I_j
\]

with \(U\) an \(R\)-submodule of the 0-homogeneous component and any \(I_j\) a well described graded ideal of \(M_I(R)\), satisfying \(I_j \star I_k = 0\) if \(j \neq k\). Under certain conditions, the graded simplicity of an arbitrary graded hom-associative algebra \(M\) is characterized and it is shown that \(M\) is the direct sum of the family of its simple graded ideals.

Key words. Hom-associative algebra, Matrix algebra, Graded algebra, Structure theory.

AMS subject classifications. 16W50, 16S35, 16S80.

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