STRUCTURED CANONICAL FORMS FOR PRODUCTS OF (SKEW-)SYMMETRIC MATRICES AND THE MATRIX EQUATION $XAX = B^*$

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Abstract. The contragredient transformation $A \mapsto P^{-1}AP^{-T}$, $B \mapsto P^TBP$ of two matrices $A, B$ effects simultaneous similarity transformations of the products $AB$ and $BA$. This work provides structured canonical forms under this transformation for symmetric or skew-symmetric $A, B$. As an application, these forms are used to study the quadratic matrix equation $XAX = B$, where both $A, B$ are skew-symmetric or symmetric matrices. Necessary and sufficient conditions for the existence of a (nonsingular) symmetric solution $X$ are formulated in terms of the structured canonical form.

Key words. Skew-symmetric matrix, Symmetric matrix, Matrix product, Quadratic matrix equation.

AMS subject classifications. 15A21, 15A24, 15B57.

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