ACCURATE COMPUTATIONS WITH TOTALLY POSITIVE BERNSTEIN–VANDERMONDE MATRICES

ANA MARCO and JOSÉ-JAVIER MARTÍNEZ

Abstract. The accurate solution of some of the main problems in numerical linear algebra (linear system solving, eigenvalue computation, singular value computation and the least squares problem) for a totally positive Bernstein–Vandermonde matrix is considered. Bernstein–Vandermonde matrices are a generalization of Vandermonde matrices arising when considering for the space of the algebraic polynomials of degree less than or equal to $n$ the Bernstein basis instead of the monomial basis.

The approach in this paper is based on the computation of the bidiagonal factorization of a totally positive Bernstein–Vandermonde matrix or of its inverse. The explicit expressions obtained for the determinants involved in the process make the algorithm both fast and accurate. The error analysis of this algorithm for computing this bidiagonal factorization and the perturbation theory for the bidiagonal factorization of totally positive Bernstein–Vandermonde matrices are also carried out.

Several applications of the computation with this type of matrices are also pointed out.

Key words. Bernstein–Vandermonde matrix, Bernstein basis, Total positivity, Neville elimination, Bidiagonal decomposition, High relative accuracy.

AMS subject classifications. 15A18, 15A23, 65D05, 65F15, 65F35.