DERIVATIVES OF TENSOR POWERS AND THEIR NORMS

RAJENDRA BHATIA†, PRIYANKA GROVER†, AND TANVI JAIN†

Abstract. The norm of the $m$th derivative of the map that takes an operator to its $k$th antisymmetric tensor power is evaluated. The case $m = 1$ has been studied earlier by Bhatia and Friedland [R. Bhatia and S. Friedland. Variation of Grassman powers and spectra. Linear Algebra and its Applications, 40:1–18, 1981]. For this purpose a multilinear version of a theorem of Russo and Dye is proved: it is shown that a positive $m$-linear map between $C^*$-algebras attains its norm at the $m$-tuple $(I, I, \ldots, I)$. Expressions for derivatives of the maps that take an operator to its $k$th tensor power and $k$th symmetric tensor power are also obtained. The norms of these derivatives are computed. Derivatives of the map taking a matrix to its permanent are also evaluated.

Key words. Determinant, Derivative, Norm, Tensor power, Antisymmetric tensor power, Symmetric tensor power, Permanent, Positive linear map, Russo-Dye theorem.

AMS subject classifications. 15A15, 15A18, 15A60, 15A69, 47A30, 47A80.