MINIMUM NUMBER OF DISTINCT EIGENVALUES OF GRAPHS∗

BAHMAN AHMADI†, FATEMEH ALINAGHIPOUR†, MICHAEL S. CAVERS‡, SHAUN FALLAT§, KAREN MEAGHER¶, AND SHAHLA NASSERASR∥

Abstract. The minimum number of distinct eigenvalues, taken over all real symmetric matrices compatible with a given graph G, is denoted by q(G). Using other parameters related to G, bounds for q(G) are proven and then applied to deduce further properties of q(G). It is shown that there is a great number of graphs G for which q(G) = 2. For some families of graphs, such as the join of a graph with itself, complete bipartite graphs, and cycles, this minimum value is obtained. Moreover, examples of graphs G are provided to show that adding and deleting edges or vertices can dramatically change the value of q(G). Finally, the set of graphs G with q(G) near the number of vertices is shown to be a subset of known families of graphs with small maximum multiplicity.

Key words. Symmetric matrix, Eigenvalue, Join of graphs, Diameter, Trees, Bipartite graph, Maximum multiplicity.

AMS subject classifications. 05C50, 15A18.

∗Received by the editors on April 2, 2013. Accepted for publication on September 7, 2013.
Handling Editor: Xingzhi Zhan.
†Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan, S4S 0A2, Canada (bahman.ahmadi@gmail.com, fatemeh.naghipour@gmail.com).
‡Department of Mathematics and Statistics, University of Calgary, Calgary, AB, T2N 1N4, Canada (michael.cavers@ucalgary.ca).
§Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan, S4S 0A2, Canada (shaun.fallat@uregina.ca). Research supported in part by an NSERC research grant.
¶Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan, S4S 0A2, Canada (karen.meagher@uregina.ca). Research supported in part by an NSERC research grant.
∥Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan, S4S 0A2, Canada (Shahla.Nasserar@uregina.ca). Research supported by PIMS and Fallat’s NSERC research grant.