CHARACTERIZING LIE ($\xi$-LIE) DERIVATIONS ON TRIANGULAR ALGEBRAS BY LOCAL ACTIONS∗

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Abstract. Let $U = \text{Tri}(A, M, B)$ be a triangular algebra, where $A, B$ are unital algebras over a field $F$ and $M$ is a faithful $(A, B)$-bimodule. Assume that $\xi \in F$ and $L : U \to U$ is a map. It is shown that, under some mild conditions, $L$ is linear and satisfies $L([X, Y]) = [L(X), Y] + [X, L(Y)]$ for any $X, Y \in U$ with $[X, Y] = XY - YX = 0$ if and only if $L(X) = \varphi(X) + ZX + f(X)$ for all $A$, where $\varphi$ is a linear derivation, $Z$ is a central element and $f$ is a central valued linear map. For the case $1 \neq \xi \in F$, $L$ is additive and satisfies $L([X, Y]_\xi) = [L(X), Y]_\xi + [X, L(Y)]_\xi$ for any $X, Y \in U$ with $[X, Y]_\xi = XY - \xi YX = 0$ if and only if $L(I)$ is in the center of $U$ and $L(A) = \varphi(A) + L(I)A$ for all $A$, where $\varphi$ is an additive derivation satisfying $\varphi(\xi A) = \xi \varphi(A)$ for each $A$. In addition, all additive maps $L$ satisfying $L([X, Y]_\xi) = [L(X), Y]_\xi + [X, L(Y)]_\xi$ for any $X, Y \in U$ with $XY = 0$ are also characterized.

Key words. Triangular algebras, Lie derivations, Derivations, $\xi$-Lie derivations, Nest algebras.

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