ON THE KEMENY CONSTANT AND STATIONARY DISTRIBUTION VECTOR FOR A MARKOV CHAIN

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Abstract. Suppose that $A$ is an irreducible stochastic matrix of order $n$, and denote its eigenvalues by $1, \lambda_2, \ldots, \lambda_n$. The Kemeny constant, $K(A)$, for the Markov chain associated with $A$ is defined as $K(A) = \sum_{j=2}^{n} \frac{1}{1-\lambda_j}$, and can be interpreted as the mean first passage from an unknown initial state to an unknown destination state in the Markov chain. Let $w$ denote the stationary distribution vector for $A$, and suppose that $w_1 \leq w_2 \leq \cdots \leq w_n$. In this paper, we show that $K(A) \geq \sum_{j=1}^{n} (j-1)w_j$, and we characterise the matrices yielding equality in that bound. The results are established using techniques from matrix theory and the theory of directed graphs.

Key words. Stochastic matrix, Stationary distribution vector, Kemeny constant.

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