THE MAXIMUM NULLITY OF A COMPLETE SUBDIVISION GRAPH IS EQUAL TO ITS ZERO FORCING NUMBER

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Abstract. Barrett et al. asked in [W. Barrett et al. Minimum rank of edge subdivisions of graphs. Electronic Journal of Linear Algebra, 18:530–563, 2009.], whether the maximum nullity is equal to the zero forcing number for all complete subdivision graphs. We prove that this equality holds. Furthermore, we compute the value of $M(F, \bar{G}) = Z(\bar{G})$ by introducing the bridge tree of a connected graph. Since this equality is valid for all fields, $\bar{G}$ has field independent minimum rank, and we also show that $\bar{G}$ has a universally optimal matrix.

Key words. Zero forcing number, Maximum nullity, Minimum rank, Complete subdivision, Bridge tree, Universally optimal, Matrix, Graph.

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