Abstract. This paper gives tight upper bound on the largest eigenvalue \( q(G) \) of the signless Laplacian of graphs with no paths of given order. Thus, let \( S_{n,k} \) be the join of a complete graph of order \( k \) and an independent set of order \( n - k \), and let \( S_{n,k}^+ \) be the graph obtained by adding an edge to \( S_{n,k} \).

The main result of the paper is the following theorem:

Let \( k \geq 1 \), \( n \geq 7k^2 \), and let \( G \) be a graph of order \( n \).

(i) If \( q(G) \geq q(S_{n,k}) \), then \( P_{2k+2} \subset G \), unless \( G = S_{n,k} \).

(ii) If \( q(G) \geq q(S_{n,k}^+) \), then \( P_{2k+3} \subset G \), unless \( G = S_{n,k}^+ \).

The main ingredient of our proof is a stability result of its own interest, about graphs with large minimum degree and with no long paths. This result extends previous work of Ali and Staton.

Key words. Signless Laplacian, Spectral radius, Forbidden paths, Stability theorem, Extremal problem.

AMS subject classifications. 05C50.