STRONG POWER AND SUBEXPONENTIAL LAWS FOR AN ORDERED LIST OF TRAJECTORIES OF A MARKOV CHAIN∗

VLADIMIR V. BOCHKAREV† AND EDUARD YU. LERNER‡

Abstract. Consider a homogeneous Markov chain with discrete time and with a finite set of states $E_0, \ldots, E_n$ such that the state $E_0$ is absorbing and states $E_1, \ldots, E_n$ are nonrecurrent. The frequencies of trajectories in this chain are studied in this paper, i.e., "words" composed of symbols $E_1, \ldots, E_n$ ending with the "space" $E_0$. Order the words according to their probabilities; denote by $p(t)$ the probability of the $t^{th}$ word in this list. As was proved recently, in the case of an infinite list of words, in the dependence of the topology of the graph of the Markov chain, there exists either the limit $\ln p(t)/\ln t$ as $t \to \infty$ or that of $\ln p(t)/t^{1/D}$, where $D \in \mathbb{N}$ (weak power and subexponential laws). As appeared, in the latter case the decreasing order of the function $p(t)$ is always subexponential (the strong subexponential law). In the first case, this paper describes necessary and sufficient conditions of the power order (the strong power law). These conditions are fulfilled, in particular, if the graph of the Markov chain that corresponds to states $E_1, \ldots, E_n$ is strongly connected.

Key words. Substochastic matrices, Markov chains, Directed graphs, Strong power laws.

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