THE (INTEGRAL) ISOMORPHISM PROBLEM, A (GEOMETRIC) GROUP THEORY POINT OF VIEW

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ABSTRACT. Given a finite group G and ring R it is natural to ask which properties of G are determined by its representations over R. The integral isomorphism problem asserts that G is fully determined by its integral group ring $\mathbb{Z}G$. False in general, it holds however for 'most' solvable and simple groups. In this talk, instead of the typical ring or module theoretical approach, we will look at above problem from a geometric group theory point of view. The question can namely be viewed as "how rigid is G inside $\mathcal{U}(\mathbb{Z}G)$?", where the latter is a finitely presented (arithmetic) group.

More precisely, after an introductory part, we will discuss the question when $\mathcal{U}(\mathbb{Z}G)$ and $GL_n(\mathcal{O})$ for $\mathcal{O}$ an order in a division algebra have property (FA). This property holds if for any action on an arbitrary (simplicial) tree there is a vertex fixed by all elements of the acting group. This is equivalent to having finite abelianisation and admitting no non-trivial decomposition as amalgamated product. In the last part of the talk, if time allows, we mention some questions and results concerning concrete constructions of amalgamated products inside $\mathcal{U}(\mathbb{Z}G)$. All results presented are joint works with Andreas Bächle, Eric Jespers, Ann Kiefer and Doryan Temmerman.