A counterexample to the first Zassenhaus conjecture

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Abstract: The research on the unit group of the integral group ring $\mathbb{Z}G$ of a finite group G was begun by Higman in 1940 and has since uncovered many interesting interactions between ring, group, representation and number theory. A conjecture of H. Zassenhaus from 1974 stated that any unit of finite order in $\mathbb{Z}G$ should be as trivial as one can possibly expect. More precisely it should be conjugate in the rational group algebra $\mathbb{Q}G$ to an element of the form $\pm g$ for some $g \in G$.

I will recall some history of the problem and then present a recently found counterexample. The existence of the counterexample is equivalent to showing the existence of a certain module over an integral group ring, which can be achieved by showing first the existence of certain modules over p-adic group rings and then considering the genus class group. These general arguments allow to boil down the question to character and group theoretic questions which can eventually be solved by mostly elementary calculations.

I am also going to present problems on finite subgroups of units in $\mathbb{Z}G$ which remain open and some results on these problems.

This is joint work with Á. del Río and F. Eisele.