

## Objectives

The conference will focus on some of the most vibrant developments in function theory related to the celebrated Whitney extension and trace problems for classes of smooth functions. These include new analytic and geometric methods in the study of differentiable structures on finite sets, extension and trace problems for functions in Sobolev spaces and spaces of generalized smoothness defined on closed subsets of  $\mathbf{R}^n$ , geometric descriptions of Sobolev extension domains, etc.

The purpose of the conference is to bring together an international group of experts in the areas of function theory and functional and geometric analysis to report on and discuss recent progress and open problems in the area of Whitney type problems and thus foster interaction and collaboration between researchers in these fields.

Motivated by boundary value problems for partial differential equations, classical trace and extension theorems characterize traces of spaces of generalized smoothness (such as Sobolev and Besov spaces etc.) to smooth submanifolds of a Euclidean space. But in many cases one needs similar results for subsets which have a more complicated geometric structure (for instance, after certain changes of variables, the initial data for a PDE may be situated on a Lipschitz surface).

This need can be seen as one of the main motivations for research in this area. This research had its origins in the seminal papers written in 1934 by Hassler Whitney. Those papers deal, in particular with the following problem: *Given a real function on an arbitrary subset of a Euclidean space, how can one determine whether it is extendable to a function of some prescribed smoothness on the whole of the space?*

Whitney developed important analytic and geometric techniques which allowed him to solve this problem for functions defined on subsets of the real line to be extended to  $m$ -times continuously differentiable functions on it. He also formulated and solved similar problems related to so-called “jets” of a function defined on a subset of a Euclidean space of any dimension. Another important result of Whitney asserts that any quasi-convex domain allows an extension of functions with bounded derivatives of a given order to functions of the same smoothness on the whole space.

In the decades since Whitney's seminal works, fundamental progress towards solving this problem and certain variants of it was made by Georges Glaeser, Yuri Brudnyi and Pavel Shvartsman, Edward Bierstone, Pierre Milman, and Wieslaw Pawlucki. Then, in a series of papers, Charles Fefferman solved the original problem of Whitney in full generality. His methods have led to a number of very important developments in the field. These include new analytic and geometric methods for the study of Lipschitz structures on finite sets, and also powerful effective algorithms (developed jointly by Charles Fefferman and Bo'az Klartag) for computation of extensions close to the best possible. Overall, there is now a much better understanding of the nature of smoothness for functions defined on very “irregular” subsets of a Euclidean space.

It is natural also to consider similar extension and trace problems for functions in Sobolev spaces. These are of interest because of their relation to the description of solutions of (linear and nonlinear) Lagrangian PDEs taking prescribed values on a general subset of  $\mathbf{R}^n$ . The research into these problems is still in a quite preliminary phase. But there are already some exciting breakthroughs. These include results about simultaneous extensions for traces of Sobolev functions obtained by Charles Fefferman jointly with Arie Israel and Kevin Luli, and also some related results obtained by Pavel Shvartsman, using very different methods.

One of the major challenges in this area is the need to find an adequate replacement of the finiteness principle of Yu. Brudnyi - P. Shvartsman for this new setting. This is very important for various problems of interpolation of experimental data. Recently Nahum Zobin found a way to construct a Hamiltonian formalism for such problems which allows one to bring powerful methods of quantum field theory into play.

A topic of central interest to the participants of this conference will deal with old problems about the description of Sobolev extension domains. After Whitney, the first deep results in this direction were obtained by Vladimir Maz'ya, Vladimir Gol'dshtein, Peter Jones, Pekka Koskela, Piotr Hajlasz, Pavel Shvartsman and Nahum Zobin. P. Shvartsman and N. Zobin have completely solved the problem for Sobolev spaces with an arbitrary order of smoothness for simply connected domains in  $\mathbf{R}^2$  and  $p > 2$ . Recently for Sobolev spaces with first order smoothness defined on planar simply connected domains in  $\mathbf{R}^2$  and  $1 < p < 2$ , the problem has been completely solved by Pekka Koskela, Tapio Rajala and Yi Zhang.