Haifa Abstracts, 2016

Hungarian-Israeli Erdős conference

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1. **The Hurwitz graph: radius, diameter, and a monoid**

Ron Adin
Bar Ilan University

The Hurwitz graph $HG(n)$ has vertices corresponding to the $(n-1)$-tuples of transpositions in the symmetric group S_n whose product is the long cycle $(1, 2, ..., n)$; its edges correspond to local left or right shifts. It has interpretations in terms of ramified covers of the Riemann sphere and in terms of flags of non-crossing partitions.

We compute the radius, and bounds on the diameter, of the Hurwitz graph. This is done by using a suitable action of the 0-Hecke monoid, leading to a well-behaved weak order. Joint work with Yuval Roichman.

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2. **Graphs, numbers and equality computation in networks**

Noga Alon
Tel Aviv University

Let G be a connected undirected graph on k vertices and suppose that on each vertex there is a player that has an input vector of n bits. The objective of the players is to decide, deterministically, whether or not all vectors are identical by sending the minimum possible number of bits along the edges of the graph. What is this minimum and what is a communication protocol achieving it ? I will describe the background of this problem focusing on a recent joint work with Efremenko and Sudakov which settles it for many (but not all) graphs. One representative result is that the minimum is $kn/2 + o(n)$ if G is Hamiltonian. The techniques combine graph theoretic ideas with tools from additive number theory.

3. **Approximately coloring graphs without long induced paths**

Maria Chudnovsky
Princeton University

It is an open problem whether the 3-coloring problem can be solved in polynomial time in the class of graphs that do not contain an induced path on t vertices, for fixed $t \geq 8$. In this talk we present a polynomial time algorithm that, given a 3-colorable graph with no induced t-vertex path constructs a coloring with at most $\max(5, t-2)$ colors. This result can also be stated as a polynomial time algorithm that given a graph G with no induced path of length t either determines that G is not 3-colorable, or outputs a coloring with at most $\max(5, t-2)$ colors. This is joint work with Oliver Schaudt, Sophie Spirkl, Maya Stein, and Mingxian Zhong.

4. **Numeration Systems and Their Uses**

Aviezri S. Fraenkel
Weizmann Institute of Science

We exhibit existence and uniqueness of several exotic numeration systems based on both the numerators and denominators of the convergents of continued fractions of irrational and rational numbers; both the greedy and lazy versions. Interesting connections between them are pointed out. We further show that they provide poly-time winning strategies for combinatorial games, analogously to data structures in computer science. Other recent uses are in the theory of partitions (number theory, joint with George Andrews and James Sellers) and phyllotaxis (abstract botany, joint with Urban Larsson).

5. **Hypergraphs without exponents**

Zoltán Füredi
Alfréd Rényi Institute of Mathematics

Let $\mathcal{H}$ be a family of k-graphs (k-uniform hypergraphs). Let $\mathbf{ex}(n, \mathcal{H})$ denote their Turán number, i.e., the maximum number of k-sets avoiding all members of $\mathcal{H}$.

Very recently Bukh, Conlon and Fox showed that given any rational number r, $1 < r < 2$, there is a finite set of graphs $\mathcal{H}$ with $\mathbf{ex}(n, \mathcal{H}) = \Theta(n^r)$. Similar statement was proved for k-uniform hypergraphs by Frankl 30 years ago.

Here we give a short, consize proof for the following result. There exists a k-uniform hypergaph H (for $k \geq 5$, k is odd) without exponent, i.e., when the Turán function is not polynomial in n. More precisely, we have $\mathbf{ex}(n, H) = o(n^{k-1})$ but it exceeds n^{k-1-c} for any positive c for $n > n_0(k, c)$.

This is an extension (and simplification) of a result of Frankl and the speaker from 1987 (where the case $k = 5$ was proven). We conjecture that it is true for all $k \geq 4$ (and probably for $k = 3$ as well).

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6. On the number of cycles when few cycle lengths are allowed.

Dániel Gerbner
Alfréd Rényi Institute of Mathematics

We are given a set L of positive integers and we consider graphs such that all their cycles have length in L. We are interested in the number of cycles the graph can contain. We give bounds for both the directed and undirected case.

Joint work with Balázs Keszegh, Cory Palmer and Balázs Patkós.

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7. The number of Hamiltonian decompositions of regular graphs.

Roman Glebov
the Hebrew University

A Hamiltonian decomposition of Γ is a partition of its edge set into disjoint Hamilton cycles. One of the oldest results in graph theory is Walecki's theorem from the 19th century, showing that a complete graph K_n on an odd number of vertices n has a Hamiltonian decomposition. This result was recently greatly extended by Kühn and Osthus. They proved that every r-regular n-vertex graph Γ with even degree $r = cn$ for some fixed $c > 1/2$ has a Hamiltonian decomposition, provided $n = n(c)$ is sufficiently large. In this talk we address the natural question of estimating $H(\Gamma)$, the number of such decompositions of Γ. The main result is that $H(\Gamma) = r^{(1+o(1))nr/2}$. In particular, the number of Hamiltonian decompositions of K_n is $n^{(1+o(1))n^2/2}$. Joint work with Zur Luria and Benny Sudakov.

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8. Hypergraph Extensions of the Erdős-Gallai Theorem

Ervin Gyõri
Alfréd Rényi Institute of Mathematics

We extend the Erdős-Gallai Theorem for Berge paths in r-uniform hypergraphs. We also find the extremal hypergraphs avoiding t-tight paths of a given length and consider this extremal problem for other definitions of paths in hypergraphs. We are to sketch the tricky proof of the Berge version consisting of three theorems in two papers. Joint papers partly with Gyula Y. Katona and Nathan Lemons, partly with Akbar Davoodi, Abhishek Methuku and Casey Tompkins are to be covered.

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9. **Bipartite algebraic graphs without quadrilaterals**

Zilin Jiang
Carnegie Mellon University

For a graph H, define the Turán number $\mathbf{ex}(n, H)$ as the maximum number of edges that a graph on n vertices can have without containing a copy of H. When H is bipartite, for example, a complete bipartite graph $K_{s,t}$, the problem of pinning down the order of magnitude of the Turán number remains in general as one of the central open problems in combinatorics. Despite the lack of progress on the Turán problem, there are certain complete bipartite graphs for which the problem has been solved asymptotically, or even exactly. The constructions that match the upper bounds in these cases are similar to one another, and they are all algebraic. In this talk, we restrict our attention to algebraic bipartite graphs defined over algebraically closed fields and we shall discuss a structural theory of those graphs that do not contain $K_{s,t}$ as a subgraph.

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10. **Topological properties of graphs and hypergraphs**

Gil Kalai
the Hebrew university of Jerusalem

Following the recent interest in "high dimensional combinatorics" we will discuss some topological views on some combinatorial questions regarding graphs and hypergraphs.

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11. **A two-part Erdős-Ko-Rado theorem**

Gyula O.H. Katona
Alfréd Rényi Institute of Mathematics

Let $|X_1| = n_1$ and $|X_2| = n_2$ be two disjoint sets and use the notation $n = n_1 + n_2$. The integers $k \leq \frac{n_1}{2}, \ell \leq \frac{n_2}{2}$ are given. Consider the subsets of $X_1 \cup X_2$ of whose intersections with X_1 (respectively X_2) have k (resp. ℓ) elements. We determine the largest intersecting subfamily of this family of subsets.

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12. **On the number of edge-disjoint triangles in K_4-free graphs**

Balázs Keszegh
Alfréd Rényi Institute of Mathematics

We prove the quarter of a century old conjecture that every K_4-free graph with n vertices and $\lfloor n^2/4 \rfloor + m$ edges contains m pairwise edge disjoint triangles. Joint work with Ervin Győri.

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13. **Client-Waiter games**

Michael Krivelevich
Tel Aviv University

For a graph G, a monotone increasing graph property P, and positive integer q, we define the Client-Waiter game (also called the Chooser-Picker game) to be a two-player game which runs as follows. In each round Waiter offers Client a subset of at least one and at most $q+1$ unclaimed edges of G. Client claims one of the offered edges, with the rest going to Waiter. The game ends when all the edges have been claimed. If Client's graph has the target property P by the end of the game, then he wins the game, otherwise Waiter is the winner.

We will discuss this general setup and the technical tools available to analyze these games. Then we present recent results for the cases when the board G is the complete graph K_n or the random graph $G(n,p)$.

Based on joint works with Dan Hefetz and Wei En Tan, and with Oren Dean.

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14. **Quadratic residues and difference sets**

Seva Lev
Oranim

It has been conjectured by Sárközy that with finitely many exceptions, the set of quadratic residues modulo a prime p cannot be represented as a sumset $\{a+b\colon a \in A, b \in B\}$ with non-singleton $A, B \subset F_p$. The case $A = B$ of this conjecture has been established by Shkredov. The analogous problem for differences remains open: is it true that for all sufficiently large primes p, the set of quadratic residues modulo p is not of the form $\{a'-a''\colon a', a'' \in A,\ a' \neq a''\}$ with $A \subset F_p$? We present the results of our recent paper, joint with Jack Sonn, where a presumably more tractable variant of this problem is considered: is there a set $A \subset F_p$ such that every quadratic residue has a *unique* representation as $a' - a''$ with $a', a'' \in A$, and no non-residue is representable in this form? We give a number of necessary conditions for the existence of such A, involving for the most part the behavior of primes dividing $p-1$. These conditions enabled us to rule out all primes p bigger than 13 and smaller than 10^{20} (the primes $p=5$ and $p=13$ being conjecturally the only exceptions).

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15. **"High-dimensional Erdős-Szekeres and Ulam's Problem"**

Nati Linial
The Hebrew University of Jerusalem

This is part of our ongoing effort to develop what we call "High-dimensional combinatorics". We equate a permutation with its permutation matrix, namely an $n \times n$ array of zeros and ones in which every line (row or column) contains exactly one 1. In analogy, a two-dimensional permutation is an $n \times n \times n$ array of zeros and ones in which every line (row, column or shaft) contains exactly one 1. It is not hard to see that a two-dimensional permutation is synonymous with a Latin square. It should be clear what a d-dimensional permutation is, and those are still very partially understood. We have already made good progress on several aspects of this field. We largely start from a familiar phenomenon in the study of permutations and seek its high dimensional counterparts. Specifically we have already made some progress on the following:

-The enumeration problem

-Birkhoff von-Neumann theorem and d-stochastic arrays

-Discrepancy phenomena

-Random generation

My main emphasis in this lecture is on monotone subsequences. A sequence of 1's in the array is considered monotone if its projection on each coordinate is a strictly monotone sequence of integers. As is well-known, a permutation in S_n has a monotone subsequence of length at least $\sqrt{n}$ and this bound is tight. There is a rich theory concerning the length of the longest increasing subsequence in a random permutation.

This part of the lecture is based on joint work with my PhD student Michael Simkin. We show that in every d-dimensional permutation there is a monotone subsequence of length $> a\sqrt{n}$ while there exist permutations with no monotone subsequence longer than $b\sqrt{n}$. Here a and b depend only on d.

On the other hand, in almost every d-dimensional permutation the longest monotone subsequence has length between $\alpha n^{d/(d+1)}$ and $\beta n^{d/(d+1)}$ where α and β depend only on d. Note that the two exponents (worst case and typical case) coincide only for $d = 1$.

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16. **The vertex and edge sign balances of (hyper)graphs**

Dezsõ Miklós
Alfréd Rényi Institute of Mathematics

Pokrovskiy and Alon, Huang and Sudakov introduced the MMS (Manickam-Miklos-Singhi) property of hypegraphs: "for every assignment of weights to its vertices with nonnegative sum, the number of edges whose total weight is nonnegative is at least the minimum degree of H".

This immediately leads to the definition of the following hypergraph parameter: The vertex sign balance of a hypergraph is the minimum number edges whose total weight is nonnegative, where the minimum is taken over all assigments of weights to the vertices with nonnegative overall sum. The vertex sign balance is always between 0 and the minimum degree of the (hypergraph). The dual of it, the edge sign balance can be defined similarly. General and special properties (for graphs or three uniform hypergraphs) of these two parameters will be presented. In particular, the characterization of the vertex sign balance of the graphs leads to the result that the question if a (hyper)graph has the MMS property is NP-complete.

17. **Forbidden subposet problems with size restrictions**

Dániel Nagy
Eötvös Loránd University:

Upper bounds to the size of a family of subsets of an n-element set that avoids certain configurations are proved. These forbidden configurations can be described by inclusion patterns and some sets having the same size. Our results are closely related to the forbidden subposet problems, where the avoided configurations are described solely by inclusions.

18. **Generalized forbidden subposet problems**

Balázs Patkós
Alfréd Rényi Institute of Mathematics

A subposet Q' of Q is a copy of another poset P in Q if there exists a bijection $i : P \to Q'$ such that whenever $p \leq_P p'$ holds, then so does $i(p) \leq'_Q i(p')$. Let $c(P,Q)$ denote the number of copies of P in Q, and we say that Q is P-free if $c(P,Q) = 0$ holds. For any three posets P, Q, R let us denote by $La(P,Q,R)$ the maximum number of copies of P over all Q-free subposets of R, i.e. $\max\{c(P,R') : R' \subseteq R, c(Q,R') = 0\}$. This generalizes the well-studied parameter $La(n,P) = La(P_1,P,Q_n)$ where P_1 is the one element poset, and Q_n is the poset of all subsets of an n-element set ordered by inclusion. In this talk I will consider the problem of determining $La(P,Q,Q_n)$ when P and Q are small posets and consider similarities and differences of the original and the generalized forbidden subposet problem.

Joint work with Daniel Gerbner and Balazs Keszegh.

19. **A Theorem on Unit Segments on the Real Line**

Rom Pinchasi
Technion

Let $n \geq 1$ be an odd integer. For every $1 \leq i \leq n$ let $s_i = (a_i, b_i)$ be an open unit segment on the real line. Let $0 \leq \epsilon < \frac{1}{2}$ be fixed. Color by green all the points (numbers) on the real line of the form $a_i + \epsilon$ and $b_i - \epsilon$. Then there exists at least one green point that belongs to an odd number of the segments $s_1, \ldots, s_n$.

20. A modified bootstrap percolation on a random graph coupled with a lattice

Miklós Ruszinkó
Alfréd Rényi Institute of Mathematics

In this paper a random graph model $G_{\mathbb{Z}2_N,p_d}$ is introduced, which is a combination of fixed torus grid edges in $(\mathbb{Z}/N\mathbb{Z})2$ and some additional random ones. The random edges are called long, and the probability of having a long edge between vertices $u, v \in (\mathbb{Z}/N\mathbb{Z})2$ with graph distance d on the torus grid is $p_d = c/Nd$, where c is some constant. We show that, *whp*, the diameter $D(G_{\mathbb{Z}2_N,p_d}) = \Theta(\log N)$. Moreover, we consider a modified non-monotonous bootstrap percolation on $G_{\mathbb{Z}2_N,p_d}$. We prove the presence of phase transitions in mean-field approximation and provide fairly sharp bounds on the error of the critical parameters. This is a joint work of Svante Janson, Robert Kozma, Miklós Ruszinkó and Yury Sokolov.

21. How many colours are needed to colour every pentagon of a graph in five colours?

Miklós Simonovits
Alfréd Rényi Institute of Mathematics

This lecture is based on a manuscript of Erdős and Simonovits from the late 1980's.

Burr, Erdős, Graham and T. Sós [1] defined and investigated a *dual* variant of the Anti-Ramsey problems. They wrote up some of their rresults also in a second paper joint with Peter Frankl [2]. As they pointed out, one of the most interesting cases they could not settle was that of C_5.

> **The dual Anti-Ramsey problem.** Let us fix a sample graph L, and consider a (variable) graph G_n on n vertices, with
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> $$e = e(G_n) > \mathbf{ex}(n, L)$$
>
> edges. Let $\chi_S(G_n, L)$ denote the *minimum* number of colours needed to colour the edges of G_n so that no $L \subseteq G_n$ has two edges of the same colour. Determine
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> $$\chi_S(n, e, L) := \min \{\chi_S(G_n, L) \ : \ e(G_n) = e\}.$$

Here we improve several results of [1] and [2]. We shall prove, among others, that if a graph G_n has $e = \left\lfloor \frac{1}{4}n^2 \right\rfloor + 1$ edges and we colour its edges so that every $C_5 \subset G_n$ is 5–coloured, then we have to use at least $\left\lfloor \frac{n}{2} \right\rfloor + 3$ colours, if n is sufficiently large. This result is sharp.

Theorem 1. *There exists a threshold* n_0 *such that if* $n > n_0$*, and a graph* G_n *has* $\lfloor \frac{1}{4}n^2 \rfloor + 1$ *edges and we colour its edges so that every* C_5 *is 5–coloured, then we have to use at least* $\lfloor \frac{n}{2} \rfloor + 3$ *colours.*

Theorem 2. *There exists a function* $\vartheta(n) \to \infty$ *such that if* $0 < k = \binom{h}{2} < \vartheta(n)$*, then the upper bound of Theorem 4.2/[1] is sharp for* $e = \lfloor \frac{1}{4}n^2 \rfloor + k$*:*

$$\chi_S(n, e, C_5) = (h+1) \left\lfloor \frac{n}{2} \right\rfloor + k.$$

Because of the monotonicity, this implies

Theorem 3. *There exists a function* $\vartheta(n) \to \infty$ *such that if* $0 < k \le \binom{h}{2} < \vartheta(n)$*, then for* $e = \lfloor \frac{1}{4}n^2 \rfloor + k$*,*

$$\chi_S(n, e, C_5) = (h+1) \left\lfloor \frac{n}{2} \right\rfloor + k + O(\sqrt{k}).$$

We prove many further related results and many of our results are proved by the Stability Method. We have several further results in this area, e.g., in some cases we can characterize all the extremal edge-colourings, however, their formulation would require more space, and several involved constructions. So we skip them here. Altogether, mostly we restricted ourselves here to the simplest versions of our results, and left out many important related results, to be mentioned in my lecture.

References

[1] S. A. Burr, P. Erdős, R. L. Graham and Vera T. Sós: Maximal antiramsey graphs and the strong chromatic number, Journal of Graph Theory, vol 13(3), (1989) pp 163–182.

[2] S. Burr, P. Erdős, P. Frankl, R. L. Graham, V. T. Sós: Further results on maximal Anti–Ramsey graphs Proc. Kalamazoo Combin. Conf., 1989 pp 193–206

[3] P. Erdős and M. Simonovits: How many colours are needed to colour every pentagon of a graph in five colours? (manuscript, under publication)

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22. Families of Hamiltonian paths with restrictions on the pairwise unions.

Dániel Soltész

Alfréd Rényi Institute of Mathematics

This talk is about two problems of the following type. What is the maximal number of Hamiltonian paths such that the union of any two paths satisfies a certain condition?

In the first problem the condition is simple, each pairwise union should contain a triangle. Since a Hamiltonian path is a bipartite graph, and a triangle is not, it is easy to prove that the maximal number of Hamiltonian paths is at most the number of balanced bipartitions of the ground set. With I. Kovács, we managed to construct families of this size, answering a question of J. Körner, S. Messuti and G. Simonyi affirmatively.

In the second problem we have a parameter c, and the condition is that each pairwise union should have independence number at most cn. Thus when we are lowering c, our restriction is getting stronger. With R. Aharoni, we proved that there is a threshold t such that when $c < t$ we can only have a family of Hamiltonian paths of constant size for any ground set. But when $t < c$, we can have families of Hamiltonian paths of exponential size compared to the size of the ground set. We also proved non trivial lower and upper bounds on t, but the exact value of the threshold remains open.

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23. **Two-person games for covering and dominating sets**

Zsolt Tuza
Alfréd Rényi Institute of Mathematics

In recent years, several games concerning domination and transversals of graphs and hypergraphs have been introduced. We survey results and open problems, including joint works with Csilla Bujtás and Mike Henning.

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24. **On the perceptron and geometry**

Amir Yehudayoff
Technion

We shall see some applications of ideas from learning theory to questions in geometry and combinatorics.