Title: Solutions to the inexact resolvent inclusion problem with applications to nonlinear analysis and optimization

Abstract: The exact resolvent inclusion problem has various applications in nonlinear analysis and optimization theory, such as devising (proximal) algorithmic schemes aiming at minimizing convex functions and finding zeros of nonlinear operators. The inexact version of this problem allows error terms to appear and hence enables one to better deal with noise and computational errors, as well as superiorization. The issue of existence and uniqueness of solutions to this problem has neither been discussed in a comprehensive way nor in a general setting. We show that if the space is a real reflexive Banach space, the inducing function is fully Legendre (a notion introduced here), and the operator is maximally monotone, then the problem admits a unique and explicit solution. We use this result to answer, in a positive way, a fundamental question which was open for many years regarding numerous known inexact algorithmic schemes (many of them have a strongly implicit nature) in various finite and infinite dimensional settings, namely whether there exist sequences satisfying these schemes in the inexact case. Our results also imply that numerous corresponding convergence results devised for the inexact case have a genuine merit. As a byproduct we show, under simple conditions, the (Holder) continuity of the protoresolvent and the continuous dependence (stability) of the solution of the inexact resolvent inclusion problem on the initial data.

This is a joint work with Simeon Reich.