Neighborhood Oriented Graph Polynomials

Peter Tittmann
Mittweida University of Applied Sciences

Let $G = (V, E)$ be a finite simple undirected graph. The open neighborhood $N_G(v)$ of a vertex $v \in V$ is the set of all vertices that are adjacent to $v$ in $G$. The closed neighborhood of $v$ is $N_G(v) \cup \{v\}$. Analogously, we define

$$N_G(W) = \bigcup_{v \in W} N_G(v) \setminus W$$

and $N_G[W] = N_G(W) \cup W$ for any vertex subset $W \subseteq V$. For a given vertex subset $W \subseteq V$, let $\partial W$ be the set of all edges of $G$ with exactly one of their end vertices in $W$, i.e.

$$\partial W = \{\{u, v\} \in E \mid u \in W, v \in V \setminus W\}.$$ 

The bipartition polynomial of $G$, introduced in [1], is

$$B(G; x, y, z) = \sum_{W \subseteq V} x^{|W|} \sum_{F \subseteq \partial W} y^{|N_G(v,F)(W)\setminus F|} z^{|F|}.$$ 

We give different representations of this polynomial and show its relations to other graph polynomials, including the domination, neighborhood, Ising, cut, independence, Eulerian subgraph, and matching polynomial.

We will discuss the role of linear orderings of the edge set, a modified version of external activity and provide some open problems.

References