Abstract

Let K be a number field and $f \in K[X]$. Carney, Hortsch and Zieve proved that the induced map $f : K \longrightarrow K$ is at most N to 1 outside of a finite set where N is the largest integer such that $\cos\left(\frac{2\pi}{N}\right) \in K$. In particular every $f \in \mathbb{Q}[X]$ is at most 6 to 1 outside of a finite set. They conjectured that for every rational map $X \longrightarrow Y$ between d dimensional varieties over a number field the map $X(K) \longrightarrow X(K)$ is at most $N(d)$ to 1 outside of a Zariski closed subvariety. The most difficult remaining open case for curves is rational functions $f : \mathbb{P}^1 \longrightarrow \mathbb{P}^1$. That is, that for every number field K there exists a constant $N(K)$ such that for any rational function $f \in K(X)$ the induced map $f : \mathbb{P}^1(K) \longrightarrow \mathbb{P}^1(K)$ is at most $N(K)$ to 1 outside of a finite set. We shall discuss advancements towards proving this conjecture.