Regular Two-Graphs and Uniform Core Graphs

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Abstract

A two-graph $(\mathcal{V}, \Delta)$ is a combinatorial entity consisting of a set $\mathcal{V}$ together with a collection Δ of unordered triples of elements of $\mathcal{V}$, such that there exists a graph G with vertex set $\mathcal{V}$ in which the triples in Δ are precisely the subgraphs K_3 and $K_2 \dot{\cup} K_1$ induced in G. Switching is an equivalence relation partitioning the graphs on n vertices into switching classes. All the graphs that are switching equivalent to G yield the same Δ and therefore the switching class of G is equivalent to its two-graph (V, Δ) in this way. Moreover, the spectrum of $(\mathcal{V}, \Delta)$ is taken to consist of the eigenvalues of the $(0, \pm 1)$-Seidel matrix which are the same for each graph in $(\mathcal{V}, \Delta)$. A regular two-graph has exactly two eigenvalues μ_1 and μ_2. An involution $\mathbf{M}(\mu_1, \mu_2)$ is used to determine the structural and combinatorial properties of the graphs of a regular two-graph. If $\mu_1 + \mu_2 = 0$, we show that $\mathbf{M}(\mu_1, \mu_2)$ is an NSSD (non-singular graph with a singular deck).

In the second part of the talk, we ask whether a benzenoid can be a core graph, that is, whether the nullity of the adjacency matrix decreases when any vertex is deleted. Moreover, can a benzenoid be a uniform core graph, that is, are there benzenoids whose nullity decrease by two when any two vertices are deleted?