Siegfried Beckus - ***Existence of periodic elements: a topological point of view***

Let $\mathcal{A}$ be a finite set and G a countable group. We consider the symbolic dynamical system $(\mathcal{A}^G, G)$ where G acts by translation on the space $\mathcal{A}^G$ of functions from G onto $\mathcal{A}$. For $K \subseteq G$ finite, elements of $\mathcal{A}^K$ are called patterns. Given a finite set of patterns, we ask for the existence of a periodic element $\omega \in \mathcal{A}^G$ such that none of the given patterns occur in ω. Or stated differently, we ask for the existence periodic configurations in a Subshift of finite type. The case $G = \mathbb{Z}$ is studied very well while this problem turned out to be hard for general countable groups. Answering this questions plays a role in the notion of chaos, whether the domino problem is decidable or undecidable and it has applications in coding theory.
In the talk, we reformulate this question in terms of topological properties of the space of invariant closed subsets of $\mathcal{A}^G$ in the Chabauty-Fell topology. Specifically, we show that existence of periodic elements in a subshift of finite type corresponds to the existence of periodic approximations in the Chabauty-Fell topology for general invariant closed subsets of $\mathcal{A}^G$.