On the horseshoe conjecture for maximal distance minimizers

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We study the properties of sets Σ having the minimal length over the class of closed connected sets $\Sigma \subset \mathbb{R}^2$ satisfying the inequality on energy

$$F_M(\Sigma) := \max_{y \in M} \operatorname{dist}(y, \Sigma) \le r$$

for a given compact set $M \subset \mathbb{R}^2$ and some given $r > 0$.

Let M be a closed convex curve with the minimal radius of curvature $R > r$. Then the connected curve Σ is called a *horseshoe*, if $F_M(\Sigma) = r$ and Σ is a union of an arc q of M_r (the shift of M on r toward the inner normal direction) with two tangent segments to M_r in the different ends of q (see Figure 1).

We prove that for every closed convex curve M with the minimal radius of curvature R and for every $r < R/5$ the set of minimizers contains only horseshoes.

Hereby we prove the conjecture of Miranda, Paolini and Stepanov describing the set of minimizers for M a circumference of radius $R > 0$ for the big enough ratio R/r.

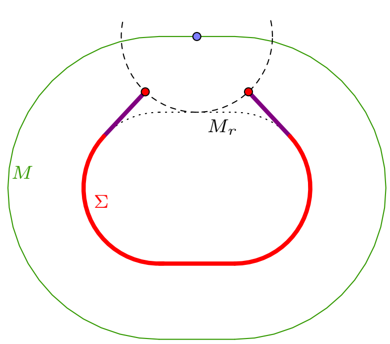

Figure 1: A horseshoe.