Quasihyperbolic metric: history and applications

Antti Rasila

Quasihyperbolic metric is a natural generalization of the hyperbolic metric, and it is closely related to the theory of quasiconformal mappings and associated function classes. The key advantage of the approach making use of this metric is that it is well suited for studying mappings in very general settings, including n-dimensional Euclidean spaces but also infinite dimensional Banach spaces and even more abstract metric spaces.

In this presentation, we give a historical overview of the quasihyperbolic metric, and outline some recent advances related to this metric and its applications in, for example, the so-called domain classification theory. We also discuss the concept of (dimension) free quasiconformality, which was introduced by J. Väisälä as a tool for generalizing the theory of quasiconformal mappings of n-dimensional Euclidean spaces into infinite dimensional Banach spaces. The concept of free quasiconformality relies heavily on the quasihyperbolic metric. We establish results demonstrating that free quasiconformality is useful in a very general metric setting. For example, we show several sufficient conditions for a homeomorphism to be fully semisolid in suitable metric spaces.

References:

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