Compactness and Structural Stability of Nonlinear Flows

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After the seminal works [1]–[3] of Brezis, Ekeland, Nayroles and Fitzpatrick, maximal monotone operators $\alpha : V \to \mathcal{P}(V')$ (V being a Banach space) and flows of the form

$$\frac{du}{dt} + \alpha(u) \ni h \quad \text{in } V', \text{ a.e. in }]0,T[, \qquad u(0) = u^0 \tag{1}$$

can be formulated as a minimization principle, even if α is not a subdifferential.

On this basis, De Giorgi's notion of Γ-convergence may be applied to the analysis of monotone inclusions. The novel notions of *structural compactness* and *structural stability* have recently been introduced, and have been applied to the Cauchy problem. In this context this keeps account of arbitrary variations not only of the datum $h \in L^2(0,T;V')$, but also of the operator α. Results rests upon the use of an exotic nonlinear topology of weak type, and on the novel notion of *evolutionary* Γ*-convergence,* [4]–[6].

The operator α may also be assumed to be a multivalued pseudo-monotone operator,

$$\alpha(u) = -\nabla \cdot \vec{\gamma}(u, \nabla u) \qquad \forall u \in W_0^{1,p}(\Omega), \tag{2}$$

with $\vec{\gamma}$ lower semicontinuous (as a multivalued operator) w.r..t. the first argument, and maximal monotone w.r..t. the second one.

These results can be extended in several directions, and can be applied to nonlinear either stationary of evolutionary PDEs, including doubly-nonlinear inclusions of the form

$$D_t \partial\varphi(u) + \alpha(u) \ni h \qquad \text{or} \qquad \alpha(D_t u) + \partial\varphi(u) \ni h, \tag{3}$$

with α a above and φ convex and lower semicontinuous.

References

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