Extremal Functions for the Singular Moser-Trudinger Inequality

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Abstract

The Moser-Trudinger embedding has been generalized by Adimurthi and Sandeep to the following weighted version: if $\Omega \subset \mathbb{R}^n$ is bounded, $\alpha > 0$ and $\beta \in [0, n)$ are such that

$$\frac{\alpha}{\alpha_n} + \frac{\beta}{n} \leq 1,$$

then

$$\sup_{\substack{v \in W_0^{1,2}(\Omega) \\ \|\nabla v\|_{L^2} \leq 1}} \int_\Omega \frac{e^{\alpha v^{\frac{n}{n-1}}} - 1}{|x|^\beta} \leq C.$$

where $\alpha_n := n\omega_{n-1}^{\frac{1}{n-1}}$ and w_{n-1} is the $(n-1)$ dimensional surface measure of the unit sphere. We prove that the supremum above is attained when $n = 2$, thus generalizing a well-known result by Flucher, who has proved the case $\beta = 0$. Major part of this talk is based on a joint work with Gyula Csató.