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In memoriam

Andrei Aleksandrovich Gonchar
November 21, 1931 – October 10, 2012

Fig. 1. A.A. Gonchar attending the International Conference on Approximation and Optimization in Havana in 1987.

Andrei Aleksandrovich Gonchar was born on November 21, 1931, in Leningrad (now St. Petersburg), Russia. He graduated from Moscow State (Lomonosov) University (MSU) in 1954. Under the supervision of S.N. Mergelyan, he defended his Candidate's (Ph.D.) thesis at the same university in 1957, and then his doctoral dissertation (D.Sc.) at the Steklov Mathematical Institute in Moscow in 1964. From 1972 until 2002 he was the head of the Department of Complex Analysis at the same Steklov Institute. He became corresponding member of the USSR Academy of Science in 1974 and was elected to full membership in 1987 (Fig. 1).

Gonchar's contributions involve a wide circle of problems in rational and harmonic approximations, convergence of continued fractions, the study of classes of quasi-analytic

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functions, asymptotic properties of orthogonal polynomials, and potential theory. Of particular impact have been his results concerning inverse type theorems in the theory of Padé approximation, the study of connections between the rate of convergence of best rational approximation and the analytic properties of the function being approximated, and his pioneering work using potential theory in the presence of an external field to obtain the asymptotic behavior of classes of orthogonal polynomials and the exact rate of convergence of best rational approximants. Gonchar was the founder of an influential scientific school in Russia. His students include several successful mathematicians with Ph.D. and D.Sc. degrees. He was a steadfast defender of classic analysts and his works were greatly influenced by them. The courses he taught at the university were exceptionally well presented and his presentations were organized with great clarity, order, and were full of ideas.

As vice president of the Russian Academy of Science from 1991 until 1998, he played a major role in trying to preserve the legendary quality of Russian mathematics by organizing, among other things, the Russian Foundation for Basic Research. From 1966 until 1988 he was deputy-Editor-in-Chief and then from 1988 until his death he was the Editor-in-Chief of Matematicheskii Sbornik which is the oldest Russian mathematics journal.

A more detailed exposition of his outstanding achievements as a researcher, scholar, and organizer of science as well as a complete list of his publications may be found in [3,1] which were published on the occasion of his 70th and 80th birthdays, respectively.

Gonchar was not only a great scientist and an excellent scholar, but also an exceptional human being. His warmth and concern for all those who surrounded him were truly remarkable. Some reminiscences on the influence of his ideas were expressed in [1]. Here, we present some additional recollections from us, his students, collaborators, and colleagues who are well known in our approximation theory community.

Alexander I. Aptekarev. Andrei Aleksandrovich Gonchar played an important role in my professional life. He opened for me research perspectives, generously providing statements of mathematical problems. He made me understand what high standards in mathematical creativity are and gave me the valuable feeling of security which one obtains by being a member of a strong mathematical school.

Formally speaking, I was not a student of Gonchar. I started my mathematical career under the supervision of Evguenii Mikhailovich Nikishin while I was a second year student at the Faculty of Physics at MSU. Nikishin was a very talented young mathematician who had done his first successful work in the area of real and harmonic analysis (Salem prize in 1973). So my work also was around those topics. By the time I graduated from the Faculty of Physics, my mentor Nikishin had changed his research interests to the study of transcendency and irrationality in number theory and naturally became motivated by Padé and Hermite-Padé rational approximation of analytic functions. These were the main topics during my graduate studies at the Faculty of Mechanics and Mathematics in MSU. During that period, I met professor Gonchar who became my mentor in the subject of Padé approximation.

In 1978–1979 Gonchar gave a one year special course on rational and Padé approximation at MSU. The activity of students and professors at that time in Mech-Math was enormous. In the listings of special courses and research seminars one could find the names of Kolmogorov, Gelfand, Sinai, Arnol'd, Novikov, and many other great mathematicians. The auditoriums were crowded with enthusiastic students. In this creative and competitive atmosphere Gonchar's special course had a great success. Over fifty undergraduate and graduate students and even some professors attended his lectures. The capacity of the auditorium was thirty seats; however, all the aisles between the desks were occupied with extra chairs from neighboring classrooms.

Indeed, it was a remarkable course. He did not use lecture notes. He had just chalk in his hands. He gave his talks with great inspiration. I never saw in my life anything similar.

In the beginning of the eighties, Gonchar and Nikishin organized a joint research seminar on convergence of Padé and Hermite-Padé rational approximants which lasted a year or a year and half. Then, the organizers decided that the seminar had achieved its goals and that there were no reasons to continue it. It was normal to have temporary research seminars. At an earlier period, Gonchar had also had a temporary seminar with Kolmogorov. The Gonchar–Nikishin seminar was very well motivated and proved to be very productive and creative. I really enjoyed very much my involvement in it. During this time I became closely associated with the remarkable scientific school of Gonchar that consisted of Evguenii Rakhmanov, Valery Vavilov, Guillermo López Lagomasino, Sergey Suetin, Ralitza Kovacheva, Viktor Buslaev, and Vasilii Prokhorov. Later, I was lucky to be able to add to this list of my very good friends and colleagues two younger students of Gonchar, Vladimir Buyarov who was, in fact, his grandstudent, and Andrei Martínez Finkelshtein. People from this list acquainted me with other personal mathematical friends. Thus, I am very indebted to Gonchar who stands at the root of this huge graph-tree.

In 1986, Evguenii Nikishin passed away and Gonchar invited me to participate on a permanent basis in the work of his seminar at the Steklov Institute. Thus I, and later two more students of Nikishin, Valeri Kaliaguine and Vladimir Sorokin, joined Gonchar's school. The creative force of Gonchar and his team was extremely strong. His ability as a top mathematical analyst in the USSR and a number of other favorable factors allowed to crack many deep problems in the theory of rational approximations of analytic functions. I personally benefited tremendously working in the environment of Gonchar's seminar and school. During that period, I worked on the strong asymptotics of Hermite-Padé approximants of Angelesco systems. This work required the study of strong asymptotics for polynomials orthogonal with respect to varying weights. I was happy to be in permanent contact with Evguenii Rakhmanov who also developed this technique for attacking strong asymptotics for orthogonal polynomials with respect to Freud weights. When my work was finished, I gave three or four talks in a row in our seminar with a storm of discussions headed by Gonchar, analyzing each part of my rather complicated proof and, finally, after approving my result, he warmly congratulated me. I have to say that his support on that result and his attention during my further mathematical work have encouraged and influenced me ever since.

Ralitza K. Kovacheva. I came to know Professor Gonchar in 1975 as a Ph.D. student on the recommendation of my professors at Sofia University. Though he was not acquainted with my prior level of knowledge and my level of preparation, he treated me very respectfully. For him, I was only a young and ambitious student who was eager to solve mathematical problems, and he gave me the chance to pursue my creative interests and to carry out research under his guidance. Already at the beginning, he created a bright and friendly atmosphere of trust and benevolence which is of vital importance for any early-stage scientist in a highly professional and stimulating environment. Despite having a full schedule, he worked out an individual plan for me based on my research topic which included weekly discussions with him. The first research problem Professor Gonchar assigned to me was from the area of the rows of generalized Padé approximants. He had observed that an appropriate asymptotic behavior of the free poles of the m-th row in the table of the classical Padé approximants, associated with a power series f around zero, suffices for the continuation of f as a meromorphic function with exactly m poles, and each limit point of the free poles is a pole of f (multiplicities included). Combining this with the classical theorem of Montessus de Ballore leads to a criterion for the m-meromorphic continuation of functions analytic at zero in terms of classical Padé approximants. My task

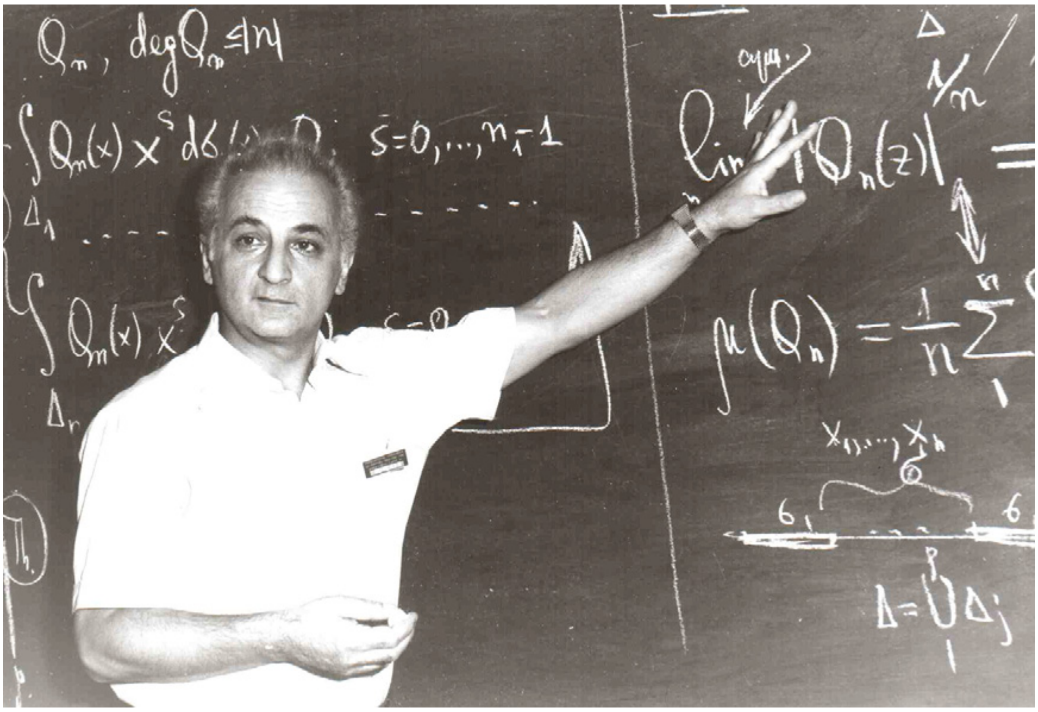

Fig. 2. A.A. Gonchar giving a talk at the International School in Complex Analysis and Application, Varna, 1983.

was to establish whether an analogous result was valid for generalized Padé approximants or, simultaneously, to point at those classes where an analog became effective. I worked on this task for months without achieving results. Though I continued working, I had already lost hope, and when I finally came to conclusions, Professor Gonchar was the first to congratulate me.

Beyond the strict boundaries of mathematics, he had an excellent general knowledge encompassing a wide range of topics from arts to international politics, to name but a few examples. I was deeply impressed by his profound knowledge of the role of the Congress of Berlin in 1878 and the political fate of the Balkans against the background of the Great Powers' rivalries. He always formed a personal opinion which was independent and not affected by extraneous factors. I still remember him saying that he'd rather read the original version of a literary work than a translation since translations tend to alter the meaning and intention of the original.

Professor Gonchar is one of the most significant persons I have ever met, in all aspects of life. I personally shall be always deeply grateful to him for providing an impetus for promising research, for having involved me in the world of mathematics and for his encouragement and support.

Guillermo López Lagomasino. I was very sad to learn of Gonchar's passing. Despite his advanced age, the image I preserve of him is one of a very energetic man full of life and ideas. The circumstances in which we met, as well as his influence on my professional career, were explained in [1]. I may add that in each of my papers there is some trace of Gonchar behind it: a concept, an idea, a trick, a result, a goal. My last paper [5], dedicated to his memory, concerns inverse type theorems for rows of type II Hermite-Padé approximation, a subject he raised in Padé approximation. The plenary talk I am preparing right now for OPSFA'12 in Tunisia deals with a Markov type theorem, another one of his favorite subjects, in the context of Hermite-Padé approximation of Nikishin systems.

Gonchar visited me twice in Cuba. The second time it was in 1987 to give a plenary lecture at the International Congress on Approximation and Optimization, which I organized at Havana University. Interestingly, that was the only occasion in which all four persons who sign this obituary and Gonchar were present at the same time. In general, he had a fear of flying and used train transportation whenever possible. However, he understood that it was an excellent opportunity to interact with colleagues from the West, specially USA, with whom contact was scarce at those times and would hastily attend knowing that he would be present. His support as a member of the Scientific Committee and plenary speaker were decisive in the success of the conference and I very much appreciated it. In one of my student's house that was large and comfortable, I organized a special informal meeting with the delegates from the Soviet Union and US participants. There was some concern among the officials of the Faculty that such a meeting might be awkward. Knowing my friends from both sides, I was sure that it would be a fine party with friendly mathematical discussions and fruitful exchange of ideas. As it turned out I was right and Gonchar set the tone.

His first visit was for only a few hours, between planes, during a winter day in January of 1986 on his way to a Texas A&M conference via Cuba and Mexico; his talk in Texas is very well described by Ed Saff, see below. I received a call from Valery Vavilov in Moscow very early in the morning through a neighbor (I didn't have a phone in my house) telling me that at that precise moment Gonchar was landing in Havana. I had just bought my first car but still didn't have a driver's license, so I asked a cousin, who lived nearby, to serve as driver to go to the airport to pick him up. Upon arrival to the airport, I saw that Gonchar was already outside the terminal walking on the sidewalk. He told me that he wanted to see the sea. So, we took him to the beaches to the East of Havana: 20 miles of white sand, blue sea, and beautiful sky. To his surprise, the beach was empty though it was a weekend and asked what was the reason. I explained that in Cuba nobody went to the beach during the winter. He laughed heartily. The temperature was over 30 degrees Celsius. Then he asked if he could go for a swim, and I said of course. Knowing that he was so cautious, I took it for granted that since there was nobody in the water he might be thinking that it was not safe enough. I was mistaken, without a second thought he immediately took off his shirt, pulled down his pants, and ran into the water in his underwear. Actually, he was asking my permission because there were two ladies present (my wife and my cousin) and he didn't have the proper swimming suit. I can still see him enjoying himself in the Caribbean Sea and feel happy that I had the opportunity to offer him that pleasure. He will always remain in my mind and in my heart.

Francisco Marcellán. I met A.A. Gonchar on his visit to Spain in October 1988 to participate in a Seminar on Orthogonal Polynomials and Approximation Theory organized by Manuel Alfaro at the University of Zaragoza. The invited lecturers were, along with Gonchar, Evguenii Rakhmanov, Guillermo López, and I. The organization of the reception was amusing, to say the least. Gonchar and Rakhmanov were received on a Monday night at the airport by Rosa, my wife, acting as car driver, and my friend Jose Luis Torrea, professor at the Autonomous University of Madrid, as translator. From the airport they went straight to the hotel since on the next morning they were to meet Guillermo and me at the Chamartin train station. Simultaneously, Guillermo and I were flying from Warsaw to Madrid on a LOT carrier. Our arrival took place at midnight and though we were forced to get up very early, our 3 h trip to Zaragoza was very pleasant with Gonchar keeping us awake with his fluid conversation, and proposing to have tea and coffee.

The four days stay at Zaragoza was great mathematically and socially. Gonchar talked about rational approximation, Guillermo presented results from his Russian D.Sc. thesis, and Rakhmanov spoke on potential theoretic methods applied to the asymptotics of orthogonal polynomials supported on unbounded sets. We enjoyed the local gastronomy and the impact produced by Pinochet's failure to win the referendum that would have guaranteed his continuity in power for at least eight more years. Gonchar reacted promptly to this relevant historical event inviting all to toast with genuine Russian vodka. Manolo Alfaro edited the contents of their talks in [8]. They were very helpful to open new areas of research in Spain.

With the complicity of our Cuban friend Guillermo and Gonchar's blessing, the Hispanic–Russian ties were reinforced through an INTAS project of the European Union entitled "Constructive Complex Analysis and Density Functionals". Initially, this project involved mathematicians from the Steklov Institute (A.A. Gonchar and S.P. Suetin), the Keldysh Institute (A.I. Aptekarev), MSU (V.S. Buyarov), Belgium (Walter Van Assche), Germany (Herbert Stahl), and Spain (J.S. Dehesa, F. Marcellán, G. López, and A. Martínez). It constituted a magnificent and successful example of international scientific collaboration. On its subsequent calls new groups were incorporated from France (B. Beckermann), Portugal (A. Branquinho), Hungary (V. Totik), and Ukraine (L.B. Golinskii). The numerous papers it produced and the valuable annual meetings that took place made evident the enthusiasm this ambitious project produced in its participants. A plenary meeting of its members in February 1995 marked the startline. Gonchar, by then vice-president of the Russian Academy of Science, managed to find the time to participate and present some of his latest results. We enjoyed his company and amenable conversation during the leisure hours of this intensive meeting which allowed to deepen many personal and mathematical contacts.

Gonchar is greatly admired within the Spanish community of orthogonal polynomials. His doctoral students, Guillermo López and Andrei Martínez Finkelshtein, have contributed to consolidate it. From this point of view, we are grateful to Gonchar's teachings and legacy for it has been transmitted to his Spanish scientific "grandchildren".

Andrei Martínez Finkelshtein. I started working on my Ph.D. under the joint supervision of A.A. Gonchar and V.V. Vavilov, when Gonchar was already very much involved in the administration of the Russian Academy of Science. He was a really busy man, and my access to him was basically on Mondays, the days when he invariably conducted his seminar at the Steklov Institute. But his comments during the seminars, covering a vast portion of mathematics, as well as some minutes of individual discussions with him afterwards were a big boost of energy and provided great ideas to think about. These discussions also obeyed a ritual: you had to stroll with him at a rather energetic pace (reflecting his personality) along the Institute's corridors, with your attention split between following Gonchar's idea and trying not to miss the always abrupt turns of his promenade.

It is difficult for me to perceive Gonchar as an isolated mathematician, an "office worm" sitting and proving his theorems. He was always surrounded by people, always willing to discuss some ideas. I believe that his contributions to mathematics are at least twofold: his undoubtedly deep results, and his seminar, that became for me a great school and a melting pot of outstanding mathematicians and non-trivial human beings.

Paul Nevai. In my opinion the two most influential results in the asymptotic theory of orthogonal polynomials in the twentieth century were those of Szegő and Rakhmanov. The role played by Szegő's mentor, that is, by Pólya is well known and is well documented, see, e.g., [2, p. 13]. On the other hand, although it is well known that Gonchar inspired Rakhmanov, it is less known why and how Gonchar became interested in ratios of orthogonal polynomials. Since a picture is worth a thousand words, especially if the picture consists of actual words, let me reproduce the place where for the first time applicable ratio asymptotics appeared, see Fig. 3, and for those whose Russian is less than perfect, here it is in English as well, see Fig. 4. In short,

1. Пусть α — неотрицательная мера, удовлетворяющая условию Сеге ($\alpha \in S$), $\{L_n\}$ — последовательность полиномов от z, ортогональных относительно меры α и нормированных условием: $L_n(z) = z^n + \ldots$, $n = 0, 1, \ldots$. Пусть, как и выше, $\varphi(z) = z + \sqrt{z^2 - 1}$, $z \in D = \mathbf{C} \setminus \Delta$, $\Delta = [-1, 1]$; ветвь корня выбрана так, что $|z + \sqrt{z^2 - 1}| > 1$ для $z \in D$. Из асимптотической формулы Сеге (см. [9], стр. 305, (12.1.3), а также стр. 470) вытекает следующее соотношение:

$$\lim_{n\to\infty} \frac{L_{n+1}}{L_n}(z) = \frac{1}{2}\varphi(z), \quad z \in D; \qquad (11)$$

сходимость равномерная внутри (на компактных подмножествах) области D.

В дальнейшем мы будем опираться только на соотношение (11). Пусть $\tilde{S}$ — класс всех мер α, сосредоточенных на отрезке Δ и таких, что соответствующие ортогональные полиномы удовлетворяют соотношению (11); $\tilde{\mathscr{M}}$ — класс мероморфных функций, определяемый аналогично классу $\mathscr{M}$, но с заменой условия $\alpha \in S$ условием $\alpha \in \tilde{S}$. Тогда $S \subset \tilde{S}$, $\mathscr{M} \subset \tilde{\mathscr{M}}$. Теорема 1 будет доказана ниже для функций $f \in \tilde{\mathscr{M}}$.

Fig. 3. Math. Sb. [7, p. 614].

1. Let α be a nonnegative measure satisfying the Szegö condition ($\alpha \in S$), and let $\{L_n\}$ be a sequence of polynomials in z orthogonal relative to the measure α and normalized by the condition $L_n(z) = z^n + \cdots$, $n = 0, 1, \ldots$. As above, let $\varphi(z) = z + \sqrt{z^2 - 1}$, $z \in D = \mathbf{C} \setminus \Delta$, $\Delta = [-1, 1]$; the branch of the square root is chosen such that $|z + \sqrt{z^2 - 1}| > 1$ for $z \in D$. The asymptotic formula of Szegö (see [9], formula (12.1.3), and also the Appendix to the Russian translation, p. 470*) implies the following relation:

$$\lim_{n\to\infty} \frac{L_{n+1}}{L_n}(z) = \frac{1}{2}\varphi(z), \quad z \in D; \qquad (11)$$

convergence is uniform inside (on compact subsets of) D.

In the sequel we shall base the proof only on the relation (11). Let $\tilde{S}$ be the class of all measures α concentrated on the interval Δ and such that the corresponding orthogonal polynomials satisfy the relation (11); let $\tilde{M}$ be the class of meromorphic functions, defined analogously to the class M but with replacement of the condition $\alpha \in S$ by the condition $\alpha \in \tilde{S}$. Then $S \subset \tilde{S}$ and $M \subset \tilde{M}$. Theorem 1 will be proved below for functions $f \in \tilde{M}$.

Fig. 4. Math. USSR Sb. [7, p. 561].

Gonchar introduced the class of measures $\tilde{S}$, a subset of the class M that I introduced about the same time, for which the corresponding orthogonal polynomials have ratio asymptotics, because he noticed that his [7, Theorem 1, p. 611] on convergence of Padé approximants works for the class $\tilde{S}$ although, at that time, he had no idea what kind of measures $\tilde{S}$ consisted of. The letter S stands for Szegő and it is a subset of the Szegő class. Although I never worked with Padé approximations, this paper of Gonchar and the subsequent brilliant work by Rakhmanov played such an important role in my professional life that I am grateful to them as long as I live. I just hope that Gonchar was aware of my dedicated mission to acquaint the Western civilization with the circle of ideas around Rakhmanov's marvelous theorem; I know Jenya is.

John Nuttall. I was saddened to learn about the passing of Dr. Gonchar, and I would like to convey the sincere condolences of my wife and myself to his family.

My first contact with Andrei was in the early 1980's at a conference in Varna, Bulgaria (Fig. 2). Memories from so far back are spotty, but I distinctly recall after dinner walking back and forth in a courtyard discussing Padé approximants to meromorphic functions, a topic in which we both had an interest. A group of his associates were also at the meeting, and, during our walk, they followed us like an honor guard.

Later in the 1980's Andrei visited us for a few days in Canada, where we could catch up on developments in approximation theory. We still have a memento of that occasion in the form of an unopened bottle of vodka that he brought. We much appreciate the thought behind his gift, but we happen to be abstainers.

The highlight of my interaction with Andrei is a one week visit that we made to Moscow in August, 1989, at his invitation. Andrei and his family were most kind and hospitable. At that time, food was in short supply, with very little being available in the Academy building where we were staying. Every afternoon we had the pleasure of joining Andrei and his wife in his apartment for a hearty lunch.

We spent some time at the Institute discussing mathematics, but what I remember most vividly are several trips to places of interest in and around Moscow. One site was the Novodevichy cemetery. The most notable was a visit to the monastery in Zagorsk (now called Sergiev Posad). On that day we were accompanied by several associates, including Sergey Suetin, with whom, I am happy to say, I am still in contact.

Andrei told us something about his early life, including his time at the siege of Leningrad, and other awful events. It is to Andrei's great credit that he was able to rise above these experiences, and eventually live a productive and successful life. He is an example to all of us, who might be inclined to complain about much less trying circumstances than Andrei endured.

Vasiliy A. Prokhorov. I have in my hands nine notebooks from my student years at the university. These are the only objects I keep that I consider important from those times. They include a yearlong course, 1977–1978, given by Gonchar on complex analysis for students of the Department of Mechanics and Mathematics at MSU. My group was called #304. They also include notes on a special course on approximation theory given by Andrei Aleksandrovich dedicated to continued fractions, Padé approximation, and potential theory. Some of the notebooks, already yellow because of the years that have passed, have annotations from presentations in Gonchar's seminar at the Steklov Institute. I open the first one and on its first page I read:

- November 29, 1976. Title: "Valery Vavilov and Guillermo López, rows of Padé approximants". Valery and Guillermo discuss inverse problems for row sequences of Padé approximation and the generalization for such sequences of the Riesz–Agmon Lemma.

  Gonchar's seminar at the institute used to take place on Mondays after lunch. It was the best time for me during all the years I spent at the university.
- November 28, 1977. As a 19-year-old student in my third year at the university, I present in the research seminar at the Institute of Mathematics the proof of Hadamard's theorem on the radii of meromorphy of analytic functions.
- November 13, 1978. Ralitza Kovacheva talks about the convergence of diagonal sequences of Padé approximants whose poles lie outside the unit circle for analytic functions inside this disk. She proves uniform convergence on compact subsets of the disk centered at the origin with radius $1/\sqrt{3}$.
- November 27, 1978. Gonchar discusses Fabry's theorem and different approaches to characterizing singularities on the boundary of the disk of convergence of a power series.

There are many other dates and annotations in these notebooks which mention Evguenii Rakhmanov, Sergey Suetin, Konstantin Lungu, who were or had been students of Andrei Aleksandrovich Gonchar.

The last notes correspond to autumn of 1983, my last semester as graduate student at MSU: in total seven and a half years of my life. Andrei Aleksandrovich was a fantastic supervisor. He not only posed problems of the highest level but also gave a basis for their solution through his courses, seminars, discussions, and his constant readiness to talk about them with his students. He transmitted his energy and passion for mathematics. I recall very well that sensation. Since then, for the rest of my scientific life, I've tried to live up to his example and impress him. As a teacher I try to follow his style. His classes were deeply considered in every detail and presented with absolute clarity, beauty, and precision in their content. Perhaps these nine notebooks contain all my life as a university student: the seminar at the Steklov Institute, his special courses, and the problems that were discussed and that I wanted to solve. I am very grateful that destiny offered me the chance to be a disciple of such a great scientist and scholar.

Evguenii A. Rakhmanov. Andrei Alexandrovich Gonchar became my adviser in 1972 when I was a 4th year student at MSU. Since then, my mathematical life has been closely connected to him in one way or another. I was lucky to meet him. He was a great mathematician and a great teacher, the kind of teacher that fits well with my own personality. He knew how to encourage and motivate his students for setting the highest goals and then to do whatever it takes to reach them. And he was demanding. Because of this, maybe, he did not have a lot of students, but he devoted a lot of time to each one of them.

Above all, he was demanding of himself. He did not publish many papers. In his opinion, not all results deserve to be published. I remember his classification of problems. "Exercise" is a question for which one knows the answer (in some cases the answer was known only to him). "Problem" is a question for which one does not know the answer (such problems were almost certainly open but in many cases he did not want to take the risk of saying so). He was a brilliant lecturer, but he did not write books. I believe that he started writing some books on several occasions, but eventually he was not satisfied with the text and abandoned it.

In 1974 Andrei Alexandrovich took me on as graduate student at the Steklov Institute where he was the head of the department of complex analysis (S.N. Mergelyan had retired from this position a short time before). He was also the organizer and true leader of a small weekly seminar in the department. The seminar was absolutely informal and at the same time very well organized. Every presentation by a member of the seminar or a guest speaker turned into a common conversation of all aspects related to the talk from technical details to a general context and had no time limit. Andrei Aleksandrovich had a special gift for "envisioning the architecture of mathematical buildings, streets, and cities" and he could communicate his ideas to others. At the same time, you never lost the "main line" of reasoning of his talks. For me and many other people our seminar was a great school that determined a "mathematical identity".

Later we became collaborators. I will share one episode. We had just completed the so-called "1/9 paper" and one day I was talking to him about the fact that every zero-centered circle is an S-curve in the field $\frac{1}{2}\log|z|$. "Very well", he said, when I finished - "it's a good counter example to our theorem". It was, indeed, a counter example to one of the main theorems of the paper. We had missed a detail which was not, in fact, too important. An extra condition, that the complement of the support of the equilibrium measure be connected, had to be added to the conditions of the theorem; it is still not clear what we can say without this condition. Our collaboration meant a lot to me. I will remember and miss him.

Edward B. Saff. My admiration for Gonchar's research work stems from his keen aesthetic sense of what is beautiful mathematics. His articles on rational approximation are not only significant for the results that he obtained, but for his elegant proofs. This elegance also carried over to his lectures. I recall one of them given at an approximation theory conference at Texas A&M University where, during his one hour talk, he only filled two lines on the blackboard; his explanations were so clear and inspiring that he did not have to resort to writing formulas. It was truly one of the best mathematical lectures that I have heard. What I also admired were his willingness and efforts to acknowledge the significant works of others.

Unfortunately, Gonchar traveled little beyond Russia in his later years, so I was fortunate that our paths crossed at the CMFT meeting held in Joensuu, Finland, in 2005. During reminiscences of our previous encounters in Helsinki and Moscow, I recalled a problem he once raised dealing with external fields. Unlike the complex plane setting he so well studied, he asked the following question: A positive unit point charge in $\mathbb{R}^3$ approaching from infinity a perfectly spherical isolated conductor (the unit sphere S^2) carrying a total charge of $+1$ will eventually cause a negatively charged spherical cap to appear. What is the smallest distance from the point charge to the sphere where still all of the sphere is positively charged? The rather surprising answer we discovered some years later turned out to be the Golden ratio $(1+\sqrt{5})/2$. In generalizing this problem to the d-dimensional sphere, we discovered an unusual sequence of polynomials in z with integer coefficients whose real roots provided the answer to the general problem, see [4]. We dubbed these "Gonchar polynomials". As a testament to their namesake, they have rather elegant properties regarding their factorization and zeros, which are not yet fully understood.

Gonchar's contributions to rational approximation, and particularly to the use of potential theoretic methods, will long endure. He has had a profound effect on the subject and on many of us in approximation theory. I feel very fortunate to have known him. He will be long remembered.

Sergey P. Suetin. Once, around 2001–2002, during a session of Gonchar's seminar on Complex Analysis at the Steklov Institute, Sasha Aptekarev was the speaker. His presentation was on Hermite-Padé approximation of two functions. He planned to give the answer to the problem under consideration in terms of a three-sheeted Riemann surface. In order to make the presentation very comprehensive to the audience, he decided to schematically represent the Riemann surface using colored chalk (in addition to the usual white chalk that he used to write all the formulas on the blackboard, he also used red, blue, and yellow). He started to draw with the colored chalks the different trajectories which he called "threads" (he thought that would make it all simpler and clearer). The drawing turned out to be very intricate, and very rapidly with all these threads Sasha got all entangled. The audience started to complain silently, and later expressed their discomfort louder and louder and demanded from Sasha some clearer explanations.

Sasha decided to address Gonchar, who had remained silent during Sasha's presentation listening with his usual friendly smile, for some moral support. With high hopes, Sasha asked Gonchar if it was all sufficiently clear and how much had he liked his colorful drawings.

In answer to this Gonchar smiled even friendlier and said: "It turns out that I am daltonic (in current English, color blind) and cannot distinguish any color whatsoever, so from your talk I have not understood anything from the very beginning. Since I am daltonic, I was turned down in my youth from the Admiral Nakhimov Naval School". After hearing this all present started to laugh and enjoy themselves.

At first Aptekarev was a little disturbed but soon came back to himself and figured out an answer. He made a half bow to Gonchar and said "I am sure that all here present will support me if I say that in your person the Russian Navy has lost a great Admiral".

Richard S. Varga. When I think of my departed friend, Andrei A. Gonchar, I immediately think of his monumental paper of 1987, joint with E.A. Rakhmanov entitled "Equilibrium distributions and the degree of rational approximation of analytic functions", see [9]. This research gives an unbelievably beautiful solution to the "1/9-Conjecture" which is described below.

With Π_m denoting all real polynomials of degree at most m and $\Pi_{m,n}$ denoting all real rational functions $r_{m,n} = p/q$, where $p \in \Pi_m$ and $q \in \Pi_n$, then $\lambda_{m,n}$ is defined as

$$\lambda_{m,n} := \inf_{r_{m,n} \in \Pi_{m,n}} \|e^{-x} - r_{m,n}(x)\|_{L_\infty[0,+\infty)}.$$

It turns out that $\lambda_{m,n}$ is finite if and only if $0 \le m \le n$, and that, after dividing out common factors, there is a unique $\hat{r}_{m,n} = \hat{p}/\hat{q}$, with $\hat{q}$ positive on $[0, +\infty)$ such that

$$\lambda_{m,n} = \|e^{-x} - \hat{r}_{m,n}(x)\|_{L_\infty[0,+\infty)}.$$

Then, researchers from around the world began working on the behavior of the constants $\lambda_{m,n}$, using techniques from number theory, approximation theory, complex variables, and finally potential theory.

One of the first results in this area by Cody, Meinardus, and Varga [6], in 1969, determined the numbers $\{\lambda_{n,n}\}_{n=0}^{14}$ with very high precision and found that they are strictly decreasing, with $\lambda_{14,14}^{1/14} \approx 1/9.544$. Later A. Schönhage [12] in 1973 established

$$\lim_{n\to\infty} \lambda_{0,n}^{1/n} = 1/3.$$

Then, as the number of coefficients available in the rational function $r_{n,n}$ that determines $\lambda_{n,n}$ has essentially twice the number of coefficients of $r_{0,n}$ that determines $\lambda_{0,n}$, Saff and Varga [11] made the following conjecture in 1977:

$$\text{Conjecture: Is} \quad \Lambda := \lim_{n\to\infty} \lambda_{n,n}^{1/n} = 1/9? \tag{1}$$

The next big contribution in this area was by Trefethen and Gutknecht [13], in 1983, using the Carathéodory–Fejér method who showed, to 15 significant figures, that

$$\Lambda = \lim_{n\to\infty} \lambda_{n,n}^{1/n} \approx 1/9.28902549192981.$$

But subsequently, Gonchar and Rakhmanov [9], in 1987, using the latest tools from potential theory, showed that Λ of (1) can be surprisingly characterized as follows. Define $f(x) = \sum_{j=1}^{\infty} a_j x^j$, where

$$a_j := \left| \sum_{\alpha | j} (-1)^\alpha \alpha \right|, \qquad j = 1, 2, \ldots.$$

Then Λ is exactly the unique positive root of the equation

$$f(\Lambda) = 1/8,$$

so that Λ can be easily determined to as many digits as you like. This beautiful answer to this interesting question puts Professor Gonchar high on the list of world-renowned mathematicians. However, the King is dead; long live the King.

Finally, we mention that the Halphen constant [10] from 1886, almost one hundred years earlier, turns out to be exactly the above constant Λ as shown by Professor A.P. Magnus. Ah, the strange and beguiling nature of mathematics!

List of Gonchar's Ph.D. students

- Rao V. Nagisetty, India, "Some problems in the theory of harmonic function spaces", Steklov Mathematical Institute, 1968 (co-advisor Sergey Nikitovich Mergelyan).
- Konstantin N. Lungu, "Some aspects of approximation theory", Steklov Mathematical Institute, 1970.
- Tat'yana Alekseevna Leont'eva, "On representation of analytic functions by series of rational fractions", Moscow State (Lomonosov) University, 1971.
- Peter Boyadjiev, Bulgaria, "Rational interpolation of analytic functions", Moscow State (Lomonosov) University, 1972.
- Valery Vasilievich Vavilov, "Padé approximation of meromorphic functions", Moscow State (Lomonosov) University, 1977.
- Evguenii A. Rakhmanov, "Some questions on the convergence theory of diagonal Padé approximation", Steklov Mathematical Institute, 1977.
- Guillermo López Lagomasino, Cuba, "On the convergence of multipoint Padé approximation", Moscow State (Lomonosov) University, 1978.
- Ralitza K. Kovacheva, Bulgaria, "Generalized Padé approximants and meromorphic continuation of functions", Steklov Mathematical Institute, 1978.
- Sergey Pavlovich Suetin, "Problems on the convergence of the Padé–Faber approximations", Steklov Mathematical Institute, 1982.
- Vasiliy A. Prokhorov, "Rational approximation and meromorphic continuation of analytic functions", Steklov Mathematical Institute, 1985.
- Mikit Kazaryan, "Separately meromorphic functions of several complex variables", Steklov Mathematical Institute, 1986 (co-advisor Evguenii Mikhailovich Chirka).
- Le ba Kkhan' Chin', North Vietnam, "Inverse theorems for multipoint Padé approximants", Moscow State (Lomonosov) University, 1988.
- Andrei Martínez Finkelshtein, Cuba, "Equilibrium measures and rate of rational approximation of the exponential on the semi-axis", Moscow State (Lomonosov) University, 1991 (co-advisor Valery Vasilievich Vavilov).

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